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APPLICATION OF DIFFERENTIAL EQUATIONS IN THE STUDY OF AEROHYDROMECHANICS

АЭРОГИДРОМЕХАНИКАНЫ ИЗИЛДӨӨДӨ ДИФФЕРЕНЦИАЛДЫК ТEҢДЕМЕЛЕРДИН КОЛДОНУЛУШУ

ПРИМЕНЕНИЕ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ПРИ ИЗУЧЕНИИ АЭРОГИДРОМЕХАНИКИ

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APPLICATION OF DIFFERENTIAL EQUATIONS IN THE STUDY OF AEROHYDROMECHANICS

Abstract
The scientific report analyzes the historical origins of mathematical modeling of fluid and gas motion in aerohydrodynamics, meteorology and oceanography, from Archimedes' equations to the Navier-Stokes equations. Also, the equation of an incompressible fluid is derived and the motion of liquids and gases is described using the Euler and Lagrange equations. The differential equations of Navier-Stokes and Euler are presented, and their practical application is considered.
Keywords: aerodynamics, turbulent and laminar flow, differential equations, Euler equation, continuum equation, Reynolds number, incompressible fluid, mathematical modeling.

Аэрогидромеханиканы изилдөөдө дифференциалдык Теңдемелердин колдонулушу

Аннотация
Илимий баяндамада аэрогидродинамикада, метеорологияда жана океанологияда суюктуктардын жана газдардын кыймылын математикалык моделдөөнүн тарыхый келип чыгышы — Архимеддин теңдемелеринен тартып Навье–Стокс теңдемелерине чейин — талданат. Ошондой эле кысылбаган суюктуктун теңдемеси чыгарылып, суюктуктар менен газдардын кыймылы Эйлер жана Лагранж теңдемелери аркылуу сүрөттөлөт. Навье–Стокс жана Эйлердин дифференциалдык теңдемелери келтирилип, алардын практикалык колдонулушу каралат.

Ачкыч сөздөр: аэродинамика, турбуленттүү жана ламинардык агым, дифференциалдык теңдемелер, Эйлер теңдемеси, континуум теңдемеси, Рейнольдс саны, кысылбаган суюктук, математикалык моделдөө.

Применение дифференциальных уравнений при изучении аэрогидромеханики

Аннотация
В научном докладе анализируются исторические истоки математического моделирования движения жидкостей и газов в аэрогидродинамике, метеорологии и океанографии, от уравнений Архимеда до уравнений Навье-Стокса. Также выведено уравнение несжимаемой жидкости и описано движение жидкостей и газов с помощью уравнений Эйлера и Лагранжа. Приведены дифференциальные уравнения Навье-Стокса и Эйлера, а также рассмотрено их практическое применение.

Ключевые слова: аэродинамика, турбулентное и ламинарное, дифференциальные уравнения, уравнение Эйлера, уравнение сплошной среды, число Рейнольдса, несжимаемая жидкость, математическое моделирование.

Relevance of the study

Due to the rapid development of high-speed computing systems and methods for numerical solution of nonlinear integro-differential equations, mathematical modeling is currently one of the most effective research methods in various fields of science and technology. In this regard, mathematical modeling significantly complements, and in some cases replaces, physical experiment. Thus, mathematical modeling methods are the most effective methods for solving complex systems of nonlinear partial differential equations.

The main objectives of the research are to determine the forces of resistance, lift, stability of motion, heat exchange and characteristics of the flow of the medium around objects.

The objective of the research is to study the interaction of solids with the environment, analyze flow characteristics, identify turbulence zones, optimally design shapes and profiles, minimize energy costs and increase the efficiency of technical devices.

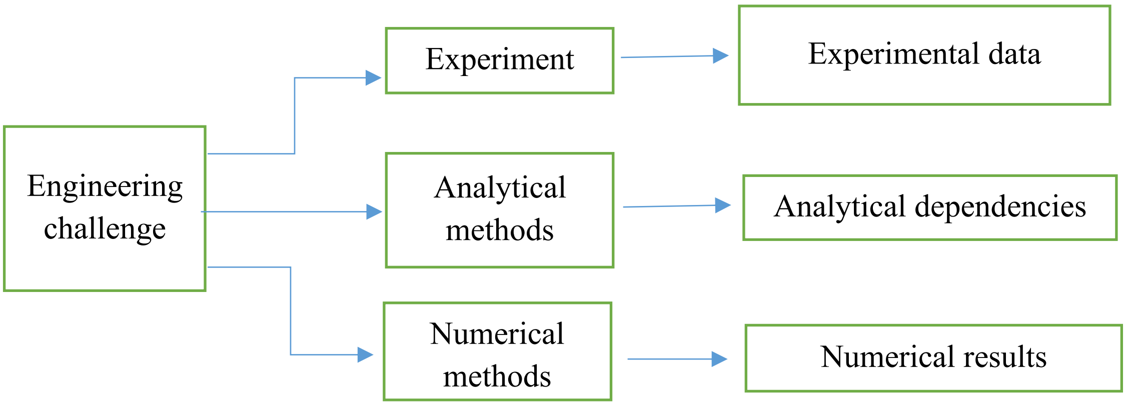
Research methods

The study of aerohydrodynamic processes is carried out using various methods, among which are theoretical, experimental and numerical approaches. In this case, theoretical and numerical methods were chosen.

The object of aerohydrodynamics research is bodies of various shapes moving in a gaseous or liquid medium, as well as fluid flows around these bodies. A fairly wide range of processes occurring in chemical and other technological processes, as well as many phenomena encountered in meteorology, oceanography, hydrodynamics and hydraulics, can be described within the framework of the Navier-Stokes equations for incompressible viscous fluids due to the low characteristic flow velocity (Ахметов & Шкадов, 2009), (Батенко & Терехов, 2002), (Белов, Исаев & Коробков, 1989), (Вабищевич, Павлов & Чурбанов, 1997), (Волков & Емельянов, 2006), (Гарбарук, Стрелец & Шур, 2012).

Mathematical modeling of liquid and gas flows is a key element in solving complex engineering problems (Патанкар, 1984). Phenomena such as vortex formation, wave propagation and mixing of liquids arise as a result of nonlinear interactions.

In the field of hydrodynamics, engineering problems are solved in the following way:

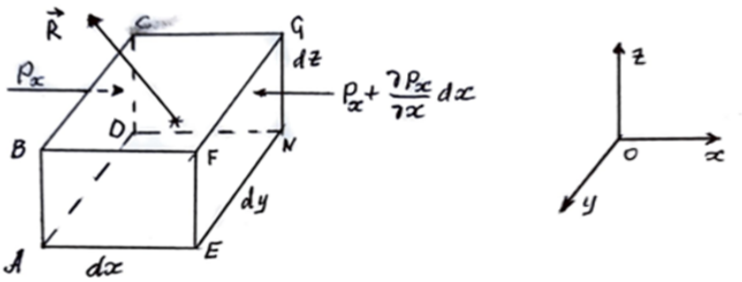


The first method appeared in humanity - this is the method of observation, i.e. experiment. Based on them, people decided hydrodynamics. Analytical methods - this is the solution of the hydrostatic equation, i.e. the Bernoulli equation. Analytical methods are one-dimensional, therefore it is necessary to solve three-dimensional cases, i.e. to build rockets and so on - numerical methods appeared. None of the methods is universal or ideal, so it considers all these three methods of solution in interrelation, only in this way it is possible to solve all the problems of hydrodynamics quite reliably. Therefore, the experimental data is corrected by analytical dependencies, and the analytical dependence is corrected by numerical methods. The obtained experimental data, analytical methods and selection by numerical methods we will finally get the result. In this article we will consider only numerical methods for solving hydrodynamic problems.

The main fundamental laws of physics are:

1. Law of conservation of mass → from it we get: Equation of continuity
2. Newton's second law → from it we get: 1. Euler equation.
   2. Navier-Stokes equation
   3. Law of conservation of energy → from it we get: Energy equation

We will prove from the Newton equation, the Euler equation, i.e. the equation of an ideal fluid. Let's take a continuous medium, i.e. an ideal fluid with a density of ρ, in it we will select an infinitely small volume in the form of a parallelepiped from the side dx dy dz (a) and the coordinate axis xyz (b), which is shown in Fig. 1.



(a) (b)

Fig.1

As we remember from the previous lecture, the equation of motion is a formulation of Newton's second law, i.e. this expression is in the following form:

$$\sum \overline{F} = m\vec{a} \qquad (1.1)$$

We need to describe what kind of force this is and how to describe mass and acceleration? That is, we need to write an equation relative to the speed of a point. Let's consider the forces that can act on a given element of liquid. This element of liquid will act - surface forces and mass forces. Surface forces are those forces that act on the surface of a body, i.e. on the surface of a parallelepiped. This can be, for example, the pressure forces of the surrounding particles that press on it with some pressure. When considering the pressure forces in an ideal liquid, we do not consider the tangential stresses, i.e. the tangential components, we consider only the normal components. Because, in a real liquid, there is no friction, so considering tangential stress is meaningless. Mass force is a force that is proportional to the mass of the body. Let's say this is the force of inertia or the forces of gravitational attraction. Therefore, let us depict or consider at the beginning the surface forces, i.e. the pressure forces. Let us designate the face ABCD on the parallelepiped. The pressure force acts on this face, which acts along the x-axis, and the center of gravity is applied to the face ABCD. This pressure P_x acts on the face ABCD. The force will be determined as the pressure acts on this face ABCD.

$P_x dy\, dz$ is the force that acts on the face of ABCD. Further, given that we consider the medium to be continuous and there are no voids in it, then we can say that the pressure functions are continuous, i.e.

$$P=f\,(x, y, z, t) \quad (1.2.)$$

Therefore, they are continuous, then it turns out that when moving to the EFGN face, we will receive the following pressure, i.e. from this side, another particle exerts pressure, which presses on the EFGN elements. But this is a different pressure. This is the pressure that was on the ABCD face plus some additional pressure.

$$P_x + \frac{\partial P_x}{\partial x} dx \qquad (1.3)$$

The force will be defined as follows:

$$P_x dy\, dz - (P_x + \frac{\partial P_x}{\partial x} dx\,) dy\, dz \qquad (1.4)$$

Formula (1.4) is the pressure force, i.e. it is a surface force.

Now let's consider the mass force. The mass force is proportional to the mass of the body applied to the center of gravity of this volume. It is applied somewhere in the center, which is shown in blue in Fig. 1. The mass force will be denoted by – ($\overrightarrow{R}$). Now we need to introduce concepts - the concepts of the intensity of the mass force, the force per unit mass, i.e. if we divide и $\vec{R}$ by dm, then we get the intensity force $\vec{F}$.

$$\overrightarrow{F_x} = \frac{\overrightarrow{R_x}}{dm} \quad (2.4) \quad \overrightarrow{R_x} = \overrightarrow{F_x} \cdot dm = \overrightarrow{F_x} \rho dxdydz \qquad (1.5)$$

$$dm = \rho dxdydx \qquad (1.6)$$

We add formula (1.5) to formula (1.4), and we get:

$$P_x dy\, dz - (P_x + \frac{\partial P_x}{\partial x} dx\,) dy\, dz + \overrightarrow{F_x} \rho dxdydz \qquad (1.7)$$

Formula (1.7) is the sum of forces acting along the ox axis on the parallelepiped. We can consider other forces in a different direction, but they have the same form. Let's rewrite formula (1.7) by opening the button and get:

$$P_x dy\, dz - P_x dy\, dz - \frac{\partial P_x}{\partial x} dxdydz + \overrightarrow{F_x} \rho dxdydz \qquad (1.8)$$

We understand that the first two members are mutually cancelled, i.e. reduced, and we will write the remaining members:

$$-\frac{\partial P_x}{\partial x} dxdydz + \overrightarrow{F_x} \rho dxdydz = \rho dxdydz \cdot \frac{dU_x}{dt} \qquad (1.9) \qquad \backslash \rho dxdydz$$

$$\begin{cases} -\dfrac{1}{\rho}\dfrac{\partial P_x}{\partial x} + F_x = \dfrac{dU_x}{dt} & (1.10) \\ -\dfrac{1}{\rho}\dfrac{\partial P_y}{\partial y} + F_y = \dfrac{dU_y}{dt} & (1.11) \\ -\dfrac{1}{\rho}\dfrac{\partial P_z}{\partial z} + F_z = \dfrac{dU_z}{dt} & (1.12) \end{cases}$$

This system is called the Euler differential equations of ideal fluid motion. This system can be written in compact form as follows:

$$-\frac{1}{\rho}\vec{\nabla} P + \vec{F} = \frac{d\vec{u}}{dt} \qquad (1.13)$$

Brief description of the differential equation of Euler motion in an ideal fluid. It is also possible to prove the continuity equation and the Navier-Stokes equation, which will result in the following mathematical differential equations:

$$-\frac{1}{\rho}\frac{dP}{dy} + \gamma\left(\frac{d^2U_y}{dx_2} + \frac{d^2U_y}{dy^2} + \frac{d^2U_y}{dz^2}\right) + F_y = \frac{dU_y}{dt} \qquad (1)$$

$$-\frac{1}{\rho}\frac{dP}{dz} + v\left(\frac{d^2z}{dx_2} + \frac{d^2U_z}{dy^2} + \frac{d^2U_z}{dz^2}\right) + F_z = \frac{dU_z}{dt} \qquad (2)$$

$$-\frac{1}{\rho}\frac{dP}{dx} + v\left(\frac{d^2U_x}{dx_2} + \frac{d^2U_x}{dy^2} + \frac{d^2U_x}{dz^2}\right) + F_x = \frac{dU_x}{dt} \qquad (3)$$

Equations (1)-(3) represent a system of equations describing the motion of a viscous real fluid, called the Navier-Stokes equations.

The current level of development of mathematics allows for the efficient use of numerical solutions of differential equations to construct realistic models of complex flows. New hybrid methods combining analytical and numerical methods are becoming widespread, allowing for accurate predictions even on large scales and under conditions of high geometric complexity.

Conclusion

Thus, the use of differential equations is an integral part of modern scientific knowledge about aero- and hydrodynamic processes and serves as the basis for efficient design and engineering calculations.

Solutions of differential equations, especially the Navier-Stokes equations, occupy an important place in the analysis of aerohydrodynamic processes. Since most practical problems are associated with nonlinear systems of complex structure, the development of efficient methods for finding solutions is a key area of scientific and technical activity.

Literature

Ахметов, В. К., Шкадов, В. Я. (2009). Численное моделирование вязких вихревых течений для технических приложений: монография. АСВ, (176).

Батенко, С. Р., Терехов, В. И. (2002). Влияние динамической предыстории потока на аэродинамику ламинарного отрывного течения в канале за обратным прямоугольным уступом. *Прикладная механика и техническая физика*, (*6*(43)), 84–92.

Белов, И. А., Исаев, С. А., Коробков, В. А. (1989). Задачи и методы расчета отрывных течений несжимаемой жидкости. Судостроение, (256).

Вабищевич, П. Н., Павлов, А. Н., Чурбанов, А. Г. (1997). Численные методы решения нестационарных уравнений Навье–Стокса в естественных переменных на частично разнесенных сетках. *Математическое моделирование*, (*9*(4)), 85–114.

Волков, К. Н., Емельянов, В. Н. (2008). Моделирование крупных вихрей в расчетах турбулентных течений. Физматлит, (368).

Гарбарук, А. В., Стрелец, М. Х., Шур, М. Л. (2012). Моделирование турбулентности в расчетах сложных течений: учебное пособие. Издательство политехнического университета, (88).

Патанкар, С. В. (1984). Численные методы решения задач теплообмена и динамики жидкости. Энергоатомиздат. (152).