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SOME PROPERTIES UNIFORM SPACE AND ITS HYPERSPACE

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Abstract. *In this paper, we will study the connection between a uniformly connected, uniformly pseudocompact, $P-$precompact and its hyperspace. It is proved that if a uniform space $(X,\mathcal{U})$ is uniformly pseudocompact iff $(\exp_c X, \exp_c \mathcal{U})$ is uniformly pseudocompact. It is also shown that if a uniform space $(X,\mathcal{U})$ is $P-$precompact, then a uniform space $(\exp_c X, \exp_c \mathcal{U})$ is $P-$precompact.*

Key words: *Hyperspace, uniformity, uniform space, uniformly connected, uniformly pseudocompact, $P-$precompact.*

НЕКОТОРОЕ СВОЙСТВА РАВНОМЕРНОЕ ПРОСТРАНСТВО И ЕГО ГИПЕРПРОСТРАНСТВО

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Аннотация. *В этой статье мы изучим связь между равномерно связным, равномерно псевдокомпактным, $P-$предкомпактом и его гиперпространством. Доказано, что если равномерное пространство $(X,\mathcal{U})$ равномерно псевдокомпактно тогда и только тогда, когда $(\exp_c X, \exp_c \mathcal{U})$ равномерно псевдокомпактно. Также показано, что если равномерное пространство $(X,\mathcal{U})$ $P-$предкомпактно, то равномерное пространство $(\exp_c X, \exp_c \mathcal{U})$ $P-$предкомпактно.*

Ключевые слова: *гиперпространство, равномерное пространство, равномерность, равномерное связное пространство, равномерное псевдокомпактное пространство, $P-$прекомпактное пространство.*

Introduction (Введение)

In [1], the connection between a finally compact, pseudocompact, extremely disconnected, $\aleph$-space and its hyperspace is studied. It is proved: if the uniform space $(X,\mathcal{U})$ is uniformly paracompact, then $(\exp_c X, \exp_c \mathcal{U})$ is uniformly paracompact, if the uniform space $(X,\mathcal{U})$ is uniformly R-paracompact, then uniform space $(\exp_c X, \exp_c \mathcal{U})$ is uniformly R-paracompact. In [2] the properties of space of the G-permutation degree, like: weight, uniform connectedness and index boundedness are studied. It was proved the G-permutation degree proserves the uniformly connected and index bounded. In the work [3] are established that the functor of idempotent probability measures with a compact support transforms open maps into open maps and preserves the weight and the completeness index of uniform spaces.

Definition 1 [4]. Let X be a nonempty set. A family $\mathcal{U}$ of coverings of a set X is called uniformity on X if the following conditions are satisfied:

(P1) If $\alpha \in \mathcal{U}$ and α is inscribed in some cover β of the set X, then $\beta \in \mathcal{U}$.

(P2) For any $\alpha_1 \in \mathcal{U}$, $\alpha_2 \in \mathcal{U}$ there exists $\alpha \in \mathcal{U}$, which is inscribed in α_1 and α_2.

(P3) For any $\alpha \in \mathcal{U}$, there exists $\beta \in \mathcal{U}$ strongly star inscribed in α.

(P4) For any x, y of a pair of different points of X, there exists $\alpha \in \mathscr{U}$ such that no element of α contains both x and y.

A family $\mathscr{U}$ consisting of a set X satisfying conditions (P1) - (P3) is called a pseudo-uniformity on X; and the pair $(X, \mathscr{U})$ is a pseudo-uniform space.

A family $\mathscr{U}$ consisting of a set X satisfying conditions (P1) - (P4) is called a uniformity on X; and the pair $(X, \mathscr{U})$ is a uniform space.

Proposition 1 [4]. For any uniformity of $\mathscr{U}$ on X, the family $\tau_{\mathscr{U}} = \{O \subset X : \text{for each } x \in O$ exists $\alpha \in \mathscr{U}$ such that $\alpha(x) \subset O\}$ is a topology on X and the topological space $(X, \tau_{\mathscr{U}})$ is a T_1-space.

The topology of $\tau_{\mathscr{U}}$ is called the topology generated or induced by the uniformity of $\mathscr{U}$.

Let $(X, \mathscr{U})$ be a uniform space and $\exp X$ the set of all nonempty closed subsets of the space $(X, \tau_{\mathscr{U}})$. For each $\alpha \in \mathscr{U}$, put $P(\alpha) = \{\langle \alpha' \rangle : \alpha' \subseteq \alpha\}$, where $\langle \alpha' \rangle = \{F \in \exp X : F \subseteq \cup \alpha' \text{ and } F \cap A \neq \varnothing \text{ for each } A \in \alpha'\}$.

Proposition 2 [8]. If $\mathscr{B}$ is the base of a uniform space $(X, \mathscr{U})$, then $P(\mathscr{B}) = \{P(\alpha) : \alpha \in \mathscr{B}\}$ forms a base of some uniformity $\exp \mathscr{U}$ on $\exp X$.

A uniform space $(\exp X, \exp \mathscr{U})$ is called a hyperspace of closed subsets of a uniform space $(X, \mathscr{U})$, and uniformity $\exp \mathscr{U}$ is called Hausdorff uniformity on $\exp X$.

Remark 1 [8]. Let $\exp_c X$ be the set of all nonempty compact subsets of the uniform space $(X, \mathscr{U})$. For each $\alpha \in \mathscr{U}$, put $K(\alpha) = \{\langle \alpha' \rangle : \alpha' \subseteq \alpha \text{ and } \alpha' \text{– finite}\}$.

Note that $K(\alpha)$ is the cover of the set $\exp_c X$.

Corollary 1 [5]. Let $(X, \mathscr{U})$ be a uniform space. Then $w(\mathscr{U}) = w(\exp \mathscr{U})$.

Corollary 2 [5]. If the uniform space $(X, \mathscr{U})$ is metrizable, then its hyperspace $(\exp X, \exp \mathscr{U})$ is also metrizable.

Theorem 1. If $(X, \mathscr{U})$ is a uniform space and $\alpha \in \mathscr{U}$ is a cover of $(X, \mathscr{U})$. Then the following equality is true $[\langle \alpha' \rangle] = \langle [\alpha'] \rangle$, where $\langle \alpha' \rangle \in P(\alpha)$ and $P(\alpha) \in \exp_c \mathscr{U}$.

A uniform space $(X, \mathscr{U})$ is called uniformly connected, and uniformity $\mathscr{U}$ is connected if any uniformly continuous mapping $f : (X, \mathscr{U}) \to (D, \mathscr{U}_D)$ of the uniform space $(X, \mathscr{U})$ into any discrete uniform space $(D, \mathscr{U}_D)$ is constant.

A finite sequence $\{A_1, A_2, ..., A_n\}$ of subsets of a set X is called linked if $A_i \cap A_{i+1} \neq \varnothing$ of each $i = 1, 2, ..., n-1$.

Definition 2 [4]. A uniform space $(X, \mathscr{U})$ is called uniformly linked if for any cover $\alpha \in \mathscr{U}$ there exists a natural number n, such that to any points $x, y \in X$ one can choose a linked sequence $\{A_1, A_2, ..., A_k\} \subset \alpha$, such that $k \leq n$, $x \in A_1$, $y \in A_k$.

Proposition 3 [4]. For a uniform space $(X, \mathscr{U})$, the following conditions are equivalent:

(1) The uniform space $(X, \mathscr{U})$ is uniformly connected.

(2) The uniformity of $\mathscr{U}$ does not contain disjoint covers consisting of at most one element.

(3) For any $\alpha \in \mathscr{U}$ and for any point $x \in X$, $\bigcup_{n=1}^{\infty} \alpha_n(x) = X$.

(4) For any $\alpha \in \mathscr{U}$ and for any points of $x, y \in X$ there exists a finite linked sequence $\{A_1, A_2, ..., A_k\} \subset \alpha,$ such that $x \in A_1,\ y \in A_k$.

Теорема 2. A uniform space $(X, \mathscr{U})$ is uniformly linked if and only if the uniform space $(\exp_c X, \exp_c \mathscr{U})$ is uniformly linked.

It follows from Proposition 3 that every uniformly linked uniform space $(X, \mathscr{U})$ is uniformly connected.

Corollary 3. A uniform space $(X, \mathscr{U})$ is uniformly connected if and only if a uniform space $(\exp_c X, \exp_c \mathscr{U})$ is uniformly connected.

Definition 3 [4]. A uniform space $(X, \mathscr{U})$ is called uniformly pseudocompact if every uniformly continuous real-valued function defined on $(X, \mathscr{U})$ is bounded.

Every Tychonoff pseudocompact space X with universal uniformity $\mathscr{U}^*$ is uniformly pseudocompact. Conversely, if a universal space $(X, \mathscr{U})$ is uniformly pseudocompact, then its topological space is pseudocompact.

A uniform space $(X, \mathscr{U})$ is uniformly pseudocompact if for every countable centered open cover $\alpha = \{V_i : i \in M\}$ of the uniform space $(X, \mathscr{U})$ the intersection $\bigcap_{i=1}^{\infty} [V_i]$ non-empty.

Theorem 3. A uniform space $(X, \mathscr{U})$ is uniformly pseudocompact if and only if a uniform space $(\exp_c X, \exp_c \mathscr{U})$ is uniformly pseudocompact.

A cover γ of a uniform space $(X, \mathscr{U})$ is said to be uniformly star-finite if there exists a uniform cover $\alpha \in \mathscr{U}$ such that $\gamma(B)$ intersects only a finite number of elements of γ for any $B \in \alpha$ [6].

A cover γ of a uniform space $(X, \mathscr{U})$ is called uniformly point-finite if for each $x \in X$ the set $\{a \in M : x \in A_a \in \gamma\}$ is finite.

Let us give examples of the property P of uniform covers of uniform spaces:

(1) covers of brevity $\leq n$;
(2) star-finite covers;
(3) point-finite covers;
(4) finite covers;
(5) covers of power $\leq \tau,\ \tau \geq \aleph_0$.

A uniform space $(X, \mathscr{U})$ is called $P-$precompact if the uniformity $\mathscr{U}$ has a base $\mathscr{B}$ consisting of covers with property P.

Theorem 4. A uniform space $(X, \mathscr{U})$ is $P-$precompact if and only if a uniform space $(\exp_c X, \exp_c \mathscr{U})$ is $P-$precompact, where properties P is a uniformly point-finite cover of uniform space.

Теорема 5. A uniform space $(X, \mathscr{U})$ is $P-$precompact if and only if a uniform space $(\exp_c X, \exp_c \mathscr{U})$ is $P-$precompact, where properties P is a uniformly star-finite cover of uniform space.

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