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The matching problem for fourth-order composite and hyperbolic equations with two lines of change of type

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Abstract. Boundary value problems for higher-order equations of mixed and mixed-composite types play a significant role in the mathematical modelling of phenomena related to heat propagation, wave processes, and the motion of weakly viscous media. The relevance of this research stems from the need for rigorous analysis of such problems, particularly in the presence of type-change lines and complex boundary conditions. The aim of the study was to formulate and comprehensively investigate a boundary value problem for a fourth-order equation of composite and hyperbolic types in a domain divided into three subdomains with differing equation structures. The problem was reduced to three auxiliary subproblems posed in the corresponding subdomains. On the lines where the type of equation changes, conjugation conditions were imposed, involving the unknown function and its derivatives up to the second order. The investigation employed classical methods from the theory of boundary value problems, techniques for order reduction, and approaches from the theory of mixed-composite type equations. Each auxiliary problem was reduced to standard formulations – namely, Dirichlet, Goursat, and Darboux problems. On the type-change lines, second-order differential equations were obtained, for which boundary value problems were solved using explicitly constructed Green's functions. The hyperbolic subproblems were reduced to Volterra and Fredholm integral equations of the second kind, and sufficient conditions for their unique solvability were derived via kernel estimates. As a result, explicit analytical expressions for the solutions in each subdomain were obtained. The results can be applied to the analysis of processes in inhomogeneous media and to the development of numerical models in mathematical physics problems

Keywords: boundary value problems; Dirichlet problem; Darboux-type problem; Green's function; matching conditions; boundary conditions; Volterra and Fredholm equations

Introduction

Boundary value problems for mixed and mixed-compound differential equations of the third and fourth orders arise in the description of various physical processes, including low-viscosity fluid flows and temperature distribution in complex media. Particularly challenging are conjugation problems, in which the values of the sought function and its derivatives are specified on lines of variation of the equation type. Such problems typically have a complex structure and require the development of special approaches for their analysis. Research in this area contributes to a deeper understanding of the properties of solutions and allows the formation of a theoretical basis for application in applied problems of mathematical physics.

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In the work of M. Mamajonov & H.M. Shermatova [1], boundary value problems for parabolic-hyperbolic equations with three lines of variation of the equation type in a triangular domain are studied. By constructing a solution, the existence and uniqueness theorem for the investigated problem is proved. K. Abdumutalip uulu [2] investigated the boundary value problem for a fourth-order partial differential equation with variable coefficients, containing the product of a mixed parabolic-hyperbolic operator and a string oscillation differential operator with discontinuous bonding conditions in a pentagon on a plane. The existence and uniqueness of the solution to the boundary value problem were proven. The solvability of this problem was reduced to solving a Fredholm integral equation of the second kind with respect to the trace of the derivative function along the line of variation of the equation. The solution to the first boundary value problem was obtained using the method of successive approximations and Green's functions. As a result, the solution to the problem was implemented by solving the Goursat problem and the first boundary value problem for the string vibration equation.

D. Amanov & O. Kilichov [3] studied boundary value problems for a fourth-order mixed-type equation in a rectangular domain and proved the existence and uniqueness of the solution to this problem. The problem under consideration differed from previously studied problems in that conjugation conditions were used instead of boundary conditions. In this case, there is no restriction on the size of the domain boundary for the solubility of the problem. If the conjugation condition is rejected, then a condition must be specified.

The boundary value problem with displacement for a third-order parabolic-hyperbolic type equation with a wave operator in the domain of hyperbolicity, when a linear combination of the values of the sought function on two independent characteristics and on the line of change is given as the boundary condition, was studied by Zh.A. Balkizov [4]. The necessary and sufficient conditions for the solvability of the problem are found.

In the work of R.R. Ashurov & M.V. Murzambetova [5], the boundary value problem for a mixed-type equation with a positive formally adjoint high-order elliptic operator is considered. In proving the theorem on the existence and uniqueness of the classical solution to the problem, where the positivity of the elliptic operator is essential. An example of a mixed-type equation with a non-negative elliptic operator is given, showing that the solution to the corresponding problem is not unique. The results of the work were obtained using the Fourier method.

Yu.P. Apakov & A.A. Sopuev [6] proved the existence of a unique solution for non-local conjugation problems in a rectangular domain for a third-order partial differential equation, where, for $y > 0$ the characteristic equation has three multiple roots, and for $y < 0$ it has one simple root and two multiple roots. Using the Green's function and the method of integral equations, the solution of the problems is equivalently reduced to the solution of the boundary value problem for the trace of the sought function at $y = 0$, and then to the solution of the Fredholm integral equation of the second kind, the solvability of which is proved by the method of successive approximations. The solution of the problem at $y > 0$ is constructed using Green's function method, and at $y < 0$ – by reducing the problem to a two-dimensional Volterra integral equation of the second kind.

Author I.A. Rudakov [7] considered the problem for the beam vibration equation, which is a fourth-order equation. The existence of an infinite number of periodic solutions of a quasi-linear equation was proven if the nonlinear term is a homogeneous odd function with power growth. The main result of the work was a theorem on the existence of a countable number of solutions to the problem. The equation considered in this work was a mathematical model of the vibrations of wires and beams.

In the work of B.Yu. Irgashev [8], a Cauchy-type problem for a high-order equation with a fractional derivative in the sense of Hilfer is considered. The existence and uniqueness theorems for the solution of the problem in the class of bounded functions constructed using automodel solutions are proved. F.M. Muminov & S.Ya. Karimov [9] investigated a mixed problem for a third-order composite equation in the multidimensional case. Using methods of generalised function theory, the necessary a priori estimates for the approximate solution of the problem were obtained, and the existence and uniqueness of a regular solution to the problem were proven. It should be noted that in the main part of the equation there were time derivatives of the Laplace operator.

From the above analysis, the formulation and study of correct problems for fourth-order composite and hyperbolic equations requires further research. The main objective of the work was to justify the correctness of the new formulation of the problem for fourth-order composite and hyperbolic equations when there are two lines of change of type in the considered domain, which are characteristics. When studying the problem, the necessity of the given conditions and gluing conditions, as well as the agreement of the given functions, was determined.

Materials and Methods

Problem statement. Let D – be the domain bounded by the lines $x = 0$, $x = -l_1$, $x = l$, $y = 0$, $y = h$, $y = -h_1$, $D = D_1 \cup D_2 \cup D_3$, $D_1 = \{x \geq 0,\ y \geq 0\}$, $D_2 = \{x \geq 0,\ y \leq 0\}$, $D_3 = \{x \leq 0,\ y > 0\}$, $(l, l_1, h, h_1 > 0)$. In the domain D, the following equations were considered:

$$\left(\frac{\partial^2}{\partial x \partial y} + a\frac{\partial}{\partial x} + b\frac{\partial}{\partial y} + c\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + pu\right) = 0, \quad (x, y) \in D_1, \qquad (1)$$

$$\frac{\partial^4 u}{\partial x^2 \partial y^2} + a_1 \frac{\partial^3 u}{\partial x^2 \partial y} + a_2 \frac{\partial^3 u}{\partial x \partial y^2} + b_1 \frac{\partial^2 u}{\partial x^2} + b_2 \frac{\partial^2 u}{\partial x \partial y} + $$
$$+ b_3 \frac{\partial^2 u}{\partial y^2} + c_1 \frac{\partial u}{\partial x} + c_2 \frac{\partial u}{\partial y} + d_1 u = 0, (x,y) \in D_2, \quad (2)$$

$$\frac{\partial^4 u}{\partial x^3 \partial y} + a_3 \frac{\partial^3 u}{\partial x^3} + a_4 \frac{\partial^3 u}{\partial x^2 \partial y} + b_4 \frac{\partial^2 u}{\partial x^2} +$$
$$+ b_5 \frac{\partial^2 u}{\partial x \partial y} + c_3 \frac{\partial u}{\partial x} + c_4 \frac{\partial u}{\partial y} + d_2 = 0, (x,y) \in D_3, \quad (3)$$

where $a_k(x, y)$, $b_i(x, y)$, $c_k(x, y)$, $d_j(x, y)$, $p(x, y)$ $(k = \overline{1,4}; i = \overline{1,5}; j = 1,2)$ – given real functions.

Equation (1) is the simplest representative of the canonical form of a composite equation with double characteristics $y = const$ and simple complex characteristics. Equations (2) and (3) are representatives of the canonical form of hyperbolic equations, since all their characteristics are real and multiple: $x = const$, $y = const$ – double for equation (2); $y = const$ – triple for equations (3).

Problem 1. Find a function $u(x, y) \in C(\bar{D}) \cap C^3 (D) \cap \cap [C^{3+1}(D_1) \cup C^{1+3}(D_1) \cup C^{2+2}(D_2) \cup C^{3+1}(D_3)]$, satisfying equations (1)-(3) in domains D_1, D_2 and D_3 respectively, and boundary conditions:

$$u(l, y) = \varphi_1(y),\ u(x, h) = \psi_1(x), 0 \le x \le l, \quad (4)$$

$$u_{xx}(l, y) = \varphi_2(y), u_{yy}(x, h) = \psi_2(x), 0 \le y \le h, \text{ in } D_1, \quad (5)$$

$$u(0, y) = \chi_1(y),\ u_x(0, y) = \chi_2(y), -h_1 \le y \le 0, \quad (6)$$

$$u(x, -h_1) = \varphi(x), 0 \le x \le l, \text{ in } D_2, \quad (7)$$

$$u(x, 0) = \chi(x), -l_1 \le x \le 0, \quad (8)$$

$$u(-l_1, y) = g_1(y),\ u_x(-l_1, y) = g_2(y), 0 \le y \le h, \text{ in } D_3, \quad (9)$$

where $\varphi_i(y)$, $\psi_i(x)$, $\chi_i(y)$, $g_i(y)$, $\varphi(x)$, $\chi(x)$ $(i = 1.2)$ – given material functions, where these functions and coefficients of equations (1)-(3) satisfy the following conditions of smoothness and compatibility:

$$a, b \in C^{1+1}(D_1);\ c, p \in C(D_1);\ a_i, c_i, d_1 \in C(\bar{D}_2),$$
$$a_{1xxy}, a_{2xyy}, b_{1xx}, b_{2xy}, b_{3yy}, c_{1x}, c_{2y} \in C(\bar{D}_1),$$
$$a_k, b_k, c_k, a_{3xxx}, a_{4xxy}, b_{3xx}, b_{4xy}, c_{3xx}, c_{4xy} \in C(\bar{D}_3), (k = 1.2),$$
$$c(x, y) \in C(D_1),\ d_1 \in C(D_2),\ d_2 \in C(D_3),$$
$$\varphi_1(y) \in C^3[0, h],\ \varphi_2(y) \in C^1[0, h],\ \psi_1(x) \in C^3[0, l],$$
$$\psi_2(x) \in C^1[0, l],\ \chi_1(y) \in C^2[-h_1, 0], \quad (10)$$

$$\chi_2(y) \in C^1[-h_1, 0],\ \varphi(x) \in C^3[0, l],\ \chi(x) \in C^2[-l_1, 0],\ g_1(y),$$
$$g_2(y) \in C^2[0, h];\ \psi_1(l) = \varphi_1(h),\ \psi_2(l) = \varphi_2(h),\ \varphi(0) = \chi_1(-h_1),$$
$$\varphi'(0) = \chi_2(-h_1),\ \chi(l_1) = g_1(0),\ g_1'(0) = g_2(-l_1),$$
$$\chi(0) = \chi_1(0),\ \chi'(0) = \chi_2(0). \quad (11)$$

From problem 1, it follows that the conditions of conjugacy are satisfied on the lines $y = 0$ and $x = 0$, respectively:

$$u(x, +0) = u(x, -0) = \tau_1(x),\ u_y(x, +0) = u_y(x, -0) =$$
$$= \nu_1(x),\ u_{yy}(x, +0) = u_{yy}(x, -0) = \mu_1(x),\ 0 \le x \le l,$$

$$u(+0, y) = u(-0, y) = \tau_2(y),\ u_x(+0, y) = u_x(-0, y) =$$
$$= \nu_2(y),\ u_{xx}(+0, y) = u_{xx}(-0, y) = \mu_2(y),\ 0 \le y \le h, \quad (12)$$

where $\tau_1(x)$, $\nu_1(x)$, $\mu_1(x)$, $\tau_2(y)$, $\nu_2(y)$, $\mu_2(y)$ – unknown functions, subject to the following matching conditions:

$$\tau_1(0) = \chi_1(0) = \chi(0),\ \tau_1(l) = \varphi_1(0),\ \tau_1''(l) =$$
$$= \varphi_2(0),\ \mu_1(l) = \varphi_1''(0),\ \nu_1(l) = \varphi_1'(0),\ \tau_2(0) = \chi(0),$$
$$\tau_2(h) = \psi_1(0),\ \nu_2(0) = \chi'(0),\ \mu_2(0) = \chi''(0),$$
$$\mu_2(h) = \psi_2(0), \nu_2(h) = \psi_1'(0).$$

After determining the functions $\tau_1(x)$, $\nu_1(x)$, $\mu_1(x)$, $\tau_2(y)$, $\nu_2(y)$, $\mu_2(y)$ problem 1 is split into the following independent auxiliary problems.

Problem 2. Find a function $u(x, y) \in C^2(\bar{D}_1) \cap [C^{3+1}) (D_1) \cup C^{1+3}(D_1)]$ satisfying equation (1) in domain D_1, boundary conditions (4), (5) and conditions:

$$u(x, +0) = \tau_1(x),\ u(+0, y) = \tau_2(y),\ 0 \le x \le l,\ 0 \le y \le h. \quad (13)$$

Problem 3. Find a function $u(x,y) \in C^1(\bar{D}_2) \cap C^{2+2}(D_2)$, satisfying equation (2) in domain D_2, boundary conditions (6), (7), and the condition:

$$u(x, -0) = \tau_1(x),\ 0 \le x \le l. \quad (14)$$

Problem 4. Find a function $u(x, y) \in C^1(\bar{D}_3) \cap C^{3+1} (D_3)$, satisfying equation (3) in domain D_3, boundary conditions (8), (9) and the condition:

$$u(-0, y) = \tau_2(y),\ 0 \le y \le h. \quad (15)$$

When solving the above problems, it is first necessary to obtain the equations on the line of change of types between individual unknown functions (12) in the corresponding areas, and then analyse them. It should be noted that the formulated tasks are solved mainly by methods of Green's and Riemann's functions, the theory of Volterra and Fredholm integral equations, as well as the principle of compressive mappings.

Results and Discussion

Equation derived from domain D_1. In the notation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + pu = z(x, y), (x, y) \in D_1. \quad (16)$$

Equations (1) for the function $z(x, y)$ can be rewritten as:

$$\frac{\partial^2 z}{\partial x \partial y} + a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} + cz = 0,\ (x, y) \in D_1, \quad (17)$$

where $z(x, y)$ – a new unknown function. For this equation, consider the Goursat problem: find a solution to equation (17) in the domain D_1 that satisfies the conditions:

$$z(x, h)=f_1(x),\ z(l, y)=f_2(y),\ 0\le x\le l,\ 0\le y\le h, \quad (18)$$

where $f_1(x)$, $f_2(y)$ – unknown functions, where $f_1(l)=f_2(h)$. The solution to problems (17) and (18) is given by formula [10]:

$$z(x,y) = R(x,h;x,y)f_1(x) + R(l,y;x,y)f_2(y) - -R(l,h;x,y)f_2(l) + \int_x^l B(t,h)\,R(t,h;x,y) - -\frac{\partial}{\partial t}R(t,h;x,y)f_2(t)dt + +\int_y^h \left(A(l,t_1)R(t_1,l;x,y) - \frac{\partial}{\partial t_1}R(l,t_1;x,y)\right)\times \times f_2(t_1)dt_1, \quad (19)$$

where $R(t, t_1; x, y)$ – Riemann function. Furthermore, substituting (19) into the right-hand side of (16) for $y=h$ and $x=1$, also taking into account boundary conditions (4), (5) for determining the function $f_1(x)$, $f_2(y)$ the following is obtained:

$$f_1(x)=\psi_1''(x)+\psi_2(x)+p\psi_1(x),$$

$$f_2(y)=\varphi_2(y)+\varphi_1''(y)+p\varphi_1(y). \quad (20)$$

Thus, the function $z(x, y)$ is determined, i.e.:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + pu = z_0(x,y), \quad (21)$$

where $z(x, y)\equiv z_0$ – a known function. Passing to the limit in (21) as $y\to+0$ and $x\to+0$ and taking into account (12), the following equations are derived from domain D_1:

$$\tau_1''(x)+\mu_1(x)+p\tau_1(x)=z_0(x, 0),\ 0\le x\le l, \quad (22)$$

$$\tau_2''(y)+\mu_2(y)+p\tau_2(y)=z_0(0, y),\ 0\le y\le h. \quad (23)$$

For equations (22) and (23) respectively, the following problems are solved:

$$1.\ \tau_1(0)=\chi_1(0),\ \tau_1(l)=\varphi_2(0). \quad (24)$$

$$2.\ \tau_2(0)=\chi(0),\ \tau_2(h)=\psi_1(0). \quad (25)$$

If equations (22) and (23) are represented as:

$$\tau_1''(x)=z_0(x, 0)-p\tau_1(x)-\mu_1(x), \quad (26)$$

$$\tau_2''(y)=z_0(0, y)-p\tau_2(y)-\mu_2(y), \quad (27)$$

then problems (26), (24) and (27), (25) will respectively be equivalent to the integral equations [11]:

$$\tau_1(x)=\int_0^l G_1(x, s)\,p(s, 0)\,\tau_1(s)ds+\alpha_1(x), \quad (28)$$

$$\tau_2(y)=\int_0^h G_2(y, s)\,p(0, s)\tau_2(s)ds+\alpha_2(y), \quad (29)$$

where
$$\alpha_1(x)=\chi_1(0)+\frac{x}{l}\left(\varphi_1(0)-\chi_1(0)\right)+ +\int_0^l G_2(x,s)\left(z_0(s,0)-\mu_1(s)\right)ds,$$

$$\alpha_2(y) = \chi(0) + \frac{y}{h}\left(\psi_1(0) - \chi(0)\right) + +\int_0^h G_1(y,s)(z_0(0,s) - \mu_2(s))ds,$$

$$G_1(x,s) = \begin{cases} \frac{x(s-l)}{l}, 0\le x < s, \\ \frac{s(x-l)}{l}, s \le x \le l, \end{cases}$$ – Green's function,

$$G_2(y,s) = \begin{cases} \frac{y(s-h)}{h}, 0\le y < s, \\ \frac{s(y-h)}{h}, s \le y \le h, \end{cases}$$ – Green's function.

Equations (28) and (29) are Fredholm integral equations of the second kind. Let:

$$l\cdot \max_{0\le x,s\le l}|pG_1| < 1,\ h\cdot \max_{0\le y,s\le h}|pG_2| < 1, \quad (30)$$

then equations (28) and (29) have unique solutions [12]:

$$\tau_1(x) = \alpha_1(x) + \int_0^l R_1(x,s)\alpha_1(s)ds, \quad (31)$$

$$\tau_2(y) = \alpha_2(y) + \int_0^h R_2(y,s)\alpha_2(s)ds, \quad (32)$$

respectively, where $R_1(x, s)$ and $R_2(y, s)$ – are the resolvent kernels of equations (28) and (29).

Equation derived from domain D_2. In the next stage, Problem 3 was considered. Due to the formulation of Problem 3 and the introduced notations (12), Goursat's problem for equation (2) with conditions (6) and $u(x, -0)=\tau_1(x)$, $u_y(x, -0)=\nu_1(x)$ is solved by formula [12]:

$$u(x,y) = A_1(x,y)\cdot\chi_1(y) - \vartheta_\eta(x,y;0,y)\chi_2(y) - -\int_0^y [B_1(x,y;\eta)\cdot\chi_2(\eta) - C_1(x,y;\eta)\chi_1(\eta)]d\eta + +\int_0^x [\vartheta(x,y;\xi,0)\nu_1''(\xi) - D_1(x,y;\xi)\tau_1''(\xi) + +a_2(\xi,0)\vartheta(x,y;\xi,0)\nu_1'(\xi) - E_1(x,y;\xi)\tau_1'(\xi) + b_3(\xi,0)\times \times\vartheta(x,y;\xi,0)\nu_1(\xi) - F_1(x,y;\xi)\tau_1(\xi)]d\xi, \quad (33)$$

where, $\vartheta(x, y; \xi, \eta)$ – the Riemann function for equation (2). Using condition (7) from (33), the equations for the functions τ_1 and ν_1 obtained from domain D_2, follow.

$$\int_0^x (\vartheta(x,-h_1;\xi,0)\nu_1''(\xi) - D_1(x,-h_1;\xi)\tau_1''(\xi) + +a_2(\xi,0)\vartheta(x,-h_1;\xi,0)\nu_1'(\xi) - -E_1(x,-h_1;\xi)\tau_1'(\xi) + b_2(\xi,0)\times \times\vartheta(x,-h_1;\xi,0)\nu_1(\xi) - F_1(x,-h_2;\xi)\tau_1(\xi))\times \times d\xi = T(x). \quad (34)$$

Carrying out integration by parts in (34), taking into account the property of the Riemann function $\vartheta(x,y;\xi,\eta)$ [13] and the compatibility conditions (11), the following is obtained:

$$D_{1\xi}(x,-h_1;x)\tau_1(x)-\vartheta_\xi(x,-h_1;x,0)\nu_1(x)-$$
$$-\int_0^x H_1(x,\xi)\tau_1(\xi)d\xi+\int_0^x H_2(x,\xi)\nu_1(\xi)d\xi=H(x),\quad(35)$$

where $H_1(x,\xi)=D_{1\xi\xi}(x,-h_1;\xi)-$
$-E_{1\xi}(x,-h_1;\xi)+F_1(x,-h_1;\xi)$,

$$H_2(x,\xi)=a_{1\xi}(\xi,0)\vartheta(x,-h_1;\xi,0)+$$
$$+a_2(\xi,0)\vartheta_\xi(x,-h_1;\xi,0)-\vartheta_{\xi\xi}(x,-h_1;\xi,0)-$$
$$-b_3(\xi,0)\vartheta(x,-h_1;\xi,0),$$

$$H(x)=T(x)+\vartheta(x,-h_1;0,0)\chi_2'(0)-$$
$$-\vartheta_\xi(x,-h_1;0,0)\chi_1'(0)-D_1(x,-h_1;0)\chi_2(0)+$$
$$+D_{1\xi}(x,-h_1;0)\chi_1(0)+a_2(0,0)\times$$
$$\times\vartheta(x,-h_1;0,0)\chi_1'(0)-E_1(x,-h_1;0)\chi_1(0).$$

On the other hand, taking into account problem 3 and aiming $y\to-0$, equation (2) can be reduced to the form:

$$\mu_1''(x)+a_1(x,0)\nu_1''(x)+a_2(x,0)\mu_1'(x)+b_1(x,0)\tau_1''(x)+$$
$$+b_2(x,0)\nu_1'(x)+b_3(x,0)\mu_1(x)+c_1(x,0)\tau_1'(x)+$$
$$+c_2(x,0)\nu_1(x)+d_1(x,0)\tau_1(x)=0.\quad(36)$$

Integrating this equation and using the aforementioned compatibility conditions, it is possible to obtain:

$$\mu_1(x)+a_1\nu_1(x)+b_1\tau_1(x)+$$
$$+\int_0^x(q_1(x,\xi)\mu_1(\xi)+q_2(x,\xi)\nu_1(\xi)+$$
$$+q_3(x,\xi)\tau_1(\xi))d\xi=f(x),\quad(37)$$

where
$$q_1(x,\xi)=a_2(\xi,0)-(x-\xi)(a_2(\xi,0)-b_3(\xi,0)),$$

$$q_2(x,\xi)=b_2(\xi,0)-2a_1(\xi,0)+(x-\xi)\times$$
$$\times\left(a_{1\xi\xi}(\xi,0)-b_2(\xi,0)+c_2(\xi,0)\right),$$

$$q_3(x,\xi)=c_1(\xi,0)-2b_1(\xi,0)+(x-\xi)\times$$
$$\times(b_1(\xi,0)-c_1(\xi,0)+d_1(\xi,0)),$$

$$f(x)=\tau_1''(0)+a_1(0,0)\tau_1'(0)+b_1(0,0)\tau_1(0)-$$
$$-b_{1x}(0,0)\tau(0)+(\nu''(0)+a_2(0,0)\tau_1''(0)+$$
$$+a_1(0,0)\nu_1'(0)-a_{1\xi}(0,0)\tau_1'(0)+b_2(0,0)\tau_1'(0)+$$
$$+b_1(0,0)\nu_1(0)+c_1(0,0)\nu_1(0))x.$$

Excluding $\tau_1(x)$ from (31) and (37), the following equation holds for the functions $\nu_1(x)$ and $\mu_1(x)$:

$$\nu_1(x)=\int_0^x H_3(x,\xi)\nu_1(\xi)d\xi+$$
$$+\int_0^l H_4(x,\xi)\mu_1(\xi)d\xi+\rho(x),\quad(38)$$

where

$$H_3(x,\xi)=-\frac{H_2(x,\xi)}{\vartheta_\xi(x,-h_1;x,0)},$$

$$H_4(x,\xi)=-\frac{1}{\vartheta_\xi(x,-h_1;x,0)}\cdot\left(D_{1\xi}(x,-h_1;\xi)R_{10}(x,\xi)-\right.$$
$$\left.-\int_\xi^x H_1(x,\xi_1)R_{10}(\xi_1,\xi)d\xi_1\right),$$

$$\rho(x)=-\frac{1}{\vartheta_\xi(x,-h_1;x,0)}\cdot$$
$$\cdot\left(H(x)-D_{1\xi}(x,-h_1;x)\alpha_{11}(x)+\int_0^x H_1(\xi)\alpha_{11}(\xi)d\xi\right).$$

Equation (38) with respect to the function $\nu_1(\xi)$ is a Volterra integral equation of the second kind, and its solution can be represented by the formula:

$$\nu_1(x)=\rho_1(x)+\int_0^l K_1(x,\xi)\mu_1(\xi)d\xi,\quad(39)$$

where

$$K_1(x,\xi)=-H_3(x,\xi)-\int_0^x R_{11}(x,\xi_1)H_3(\xi_1,\xi)d\xi_1,$$

$$\rho_1(x)=-\rho(x)-\int_0^x R_{11}(x,\xi)\rho(\xi)d\xi;$$

$R_{11}(x,\xi)$ – the resolvent kernel of $H_3(x,\xi)$.

Next, substituting $\tau_1(x)$ from (31) and $\nu_1(x)$ from (39), into equation (34) yields the equation:

$$\mu_1(x)+\int_0^x q_1(x,\xi)\mu_1(\xi)d\xi+$$
$$+\int_0^l K_2(x,\xi)\mu_1(\xi)d\xi=\Phi_1(x),\quad(40)$$

where

$$K_2(x,\xi)=a_1(x,0)K_1(x,\xi)+b_1(x,0)R_{10}(x,\xi)+$$
$$+\int_0^x K_1(\xi_1,\xi)q_2(x,\xi_1)d\xi_1+\int_0^x q_3(x,\xi_1)R_{10}(\xi_1,\xi)d\xi_1,$$
$$\Phi_1(x)=f(x)-\int_0^x(q_2(x,\xi)\rho_1(\xi)+q_1(x,\xi)\alpha_{11}(\xi))d\xi.$$

Now, by solving the Volterra part of equation (40), it can be reduced to a Fredholm integral equation of the second kind:

$$\mu_1(x)+\int_0^l K(x,\xi)\mu_1(\xi)d\xi=\Phi_2(x),\quad(41)$$

where $K(x,\xi)=q_1(x,\xi)+\int_0^x R(x,\xi_1)q_1(\xi_1,\xi)d\xi_1$, $\Phi_2(x)=\Phi_1(x)+\int_0^x R(x,\xi)\Phi_1(\xi)d\xi$, $-R(x,\xi)$ the resolvent kernel of $q_1(x,\xi)$. Let

$$l\cdot N(l)<1,\quad(42)$$

where $N(l)=\max_{0\le x,\xi\le l}|K(x,\xi)|$. Then equation (41) has a unique solution. Thus, by defining the function $\mu_1(x)$ as the solution to equation (41) and substituting its value into (31) and (39), the functions $\tau_1(x)$ and $\nu_1(x)$ can be found respectively, and thereby the solution to Problem 3. The solution to Problem 3 in domain D_2 can be represented as (33).

Equation derived from domain D_3. Next, the derivation of the formula for solving Problem 4 is carried out. The solution to equation (3), satisfying the boundary conditions:

$$u(x,0)=\chi(x), u(-0,y)=\tau_2(y),$$
$$u_x(-0,y)=\nu_2(y),$$
$$u_{xx}(-0,y)=\mu_2(y),$$

is given by the formula [13]:

$$u(x,y)=F(x,y)+\int_0^y(\vartheta(x,y;0,\eta)\,\mu_2'(\eta)+a_3(0,\eta)\times \times\vartheta(x,y;0,\eta)\mu_2(\eta)-F_1(x,y;\eta)\nu_2'(\eta)-F_2(x,y;\eta)\times \times\nu_2(\eta)+F_3(x,y;\eta)\tau_2'(\eta)+F_4(x,y;\eta)\tau_2(\eta))d\eta, \quad (43)$$

where $\vartheta(x, y; \xi, \eta)$ – Riemann function for equation (3), and $F(x, y) = \vartheta_{\xi\xi}(x, y; x, 0)\,\chi(x) - \int_0^x F_5(x, y; \xi)\,\chi(\xi)d\xi$; $F_k(x, y; \eta)(k = \overline{1,5})$ – well-defined functions, which are expressed through the coefficients of equation (3) and Riemann functions $\vartheta(x, y; \xi, \eta)$. Based on formula (43), boundary conditions (9) are reduced to a system of equations:

$$\vartheta(-l_1,y;0,y)\mu_2(y)-F_1(-l_1,y,y)\nu_2(y)+ +F_3(-l_1,y,y)\tau_2(y)+\int_0^y(k_{21}(y,\eta)\mu_2(\eta)+ +k_{22}(y,\eta)\nu_2(\eta)+k_{23}(y,\eta)\tau_2(\eta))d\eta=P_1(y), \quad (44)$$

$$\vartheta_x(-l_1,y;0,y)\mu_2(y)-F_{1x}(-l_1,y,y)\nu_2(y)+ +F_{3x}(-l_1,y,y)\tau_2(y)+\int_0^y(k_{31}(y,\eta)\mu_2(\eta)+ +k_{32}(y,\eta)\nu_2(\eta)+k_{33}(y,\eta)\tau_2(\eta))d\eta=P_2(y), \quad (45)$$

where $k_{21}(y,\eta) = -\vartheta_\eta(-l_1,y;0,\eta) + a_3(0,\eta)\vartheta(-l_1,y;0,\eta)$,

$$k_{22}(y,\eta)=F_{1\eta}(-l_1,y,\eta)-F_2(-l_1,y,\eta),$$
$$k_{23}(y,\eta)=F_4(-l_1,y,y)-F_{3\eta}(-l_1,y,\eta),$$
$$k_{31}(y,\eta)=-\vartheta_{\eta x}(-l_1,y;0,\eta)+a_3(0,\eta)\vartheta_x(-l_1,y;0,\eta),$$
$$k_{32}(y,\eta)=F_{1\eta x}(-l_1,y,\eta)-F_{2\eta x}(-l_1,y,\eta),$$
$$k_{33}(y,\eta)=F_{4x}(-l_1,y,\eta)-F_{3\eta x}(-l_1,y,\eta),$$

$$P_1(y)=g_1(y)-F(-l_1,y)+F_3(-l_1,y,0)\,\tau_2(0)+\vartheta(-l_1,y;0,0)\times \times\chi''(0)-F_1(-l_1,y,0)\chi'(0),$$
$$P_2(y)=g_2(y)-\chi_x(-l_1,y)+\vartheta_x(-l_1,y;0,0)\,\chi''(0)-F_{1x}(-l_1,y,0)\times \times\chi'(0)-F_{3x}(-l_1,y,0)\chi(0).$$

Let $$\Delta=\begin{vmatrix}\vartheta(-l_1,y;0,\eta) & -F_1(-l_1,y,y)\\ \vartheta_x(-l_1,y;0,\eta) & -F_{1x}(-l_1,y,y)\end{vmatrix}\neq 0. \quad (46)$$

Equations (44) and (45) with respect to the functions $\nu_2(y)$ and $\mu_2(y)$ represent a system of Volterra integral equations of the second kind. The solution of the system (44), (45) due to condition (46) is represented by formula [11]:

$$\nu_2(y)=m_1(\tau_2(y))+\int_0^y M_1(y,s)m_1(\tau_2(s))ds, \quad (47)$$

$$\mu_2(y)=m_2(\tau_2(y))+\int_0^y M_2(y,s)m_2(\tau_2(s))ds, \quad (48)$$

where M_1, M_2 – elements of the matrix resolvent of the matrix kernel; m_1, m_2 – are well-defined functions expressed in terms of the functions $\tau_2(y)$. After substituting the value of the function $\tau_2(y)$ from (32) into the right-hand side of (48), it reduces to a Fredholm integral equation of the second kind:

$$\mu_2(y)=\int_0^h M(y,s)\mu_2(s)ds+m(y), \quad (49)$$

where $M(y, s)$, $m(y)$ – well-defined functions that are expressed through the elements of the matrix Δ^{-1} and the data of problem 1 in domains D_1 and D_3. If the condition:

$$h\cdot M(h)<1, \quad (50)$$

where $M(h)=\max_{0\le y,s\le h}|M(y,s)|$, then equation (49) has a unique solution [12]. By determining $\mu_2(y)$ from (49) and substituting its value into (32) $\tau_2(y)$ is found. After this $\nu_2(y)$, is determined from (47), thus providing the solution to Problem 4. The solution to Problem 4 in domain D_3 is defined by formula (43).

Solution to problem 1 in domain D_1. After determining the functions $\tau_1(x)$ and $\tau_2(y)$ the solution of problem 1 in the domain D_1 is determined as the solution of problem 2. In section 2, it is shown that after reducing the order, equation (1) with the solution of the Goursat's problem gives equation (21). Consequently, the solution to problem 2 is equivalent to the solution to the Dirichlet problem for equation (21) with boundary conditions (4) and $u(x, 0) = \tau_1(x)$, $u(0, y) = \tau_2(y)$, $(0 \le x \le l$, $0 \le y \le h)$. If equation (21) be rewritten as:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=z_0(x,y)-p(x,y)u, \quad (51)$$

then, using Green's function, Dirichlet's problem can be equivalently reduced to an integral equation

$$u(x,y)=\int_0^l d\xi\int_0^h G(x,y;\xi,\eta)\,p(\xi,\eta)\,u(\xi,\eta)d\eta+Q(x,y), \quad (52)$$

where
$G(x,y;\xi,\eta)=\frac{4lh}{\pi^2}\sum_{n=1}^{+\infty}\sum_{m=1}^{+\infty}\frac{1}{h^2n^2+l^2m^2}\cdot \sin\left(\frac{\pi n}{l}x\right)\cdot \sin\left(\frac{\pi m}{h}y\right)\cdot \sin\left(\frac{\pi n}{l}\xi\right)\cdot \sin\left(\frac{\pi m}{h}\eta\right)$ – Green's function of the Dirichlet problem in the domain D_1 [13] for the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$;

$$Q(x,y)=\int_0^l G_\eta(x,y;\xi,0)\tau_1(\xi)\,d\xi- -\int_0^l G_\eta(x,y;\xi,h)\psi_1(\xi)\,d\xi+\int_0^h G_\xi(x,y;0,\eta)\tau_2(\eta)\,d\eta- -\int_0^h G_\xi(x,y;l,\eta)\times\varphi_1(\eta)d\eta-\int_0^l d\xi\times \times\int_0^h G(x,y;\xi,\eta)z_0(\xi,\eta)\,d\eta.$$

Let, $$l\cdot h\cdot \max_{\substack{0\le x,\xi\le l,\\ 0\le y,\eta\le h}}\left|p(\xi,\eta)G(x,y;\xi,\eta)\right|<1. \quad (53)$$

Then equation (52) has a unique solution, which can be represented as:

$$u(x,y)=\int_0^l d\xi\int_0^h L(x,y;\xi,\eta)\,Q(\xi,\eta)d\eta+Q(x,y),$$

where $L(x,y;\xi,\eta)$ – the resolvent of the kernel equation (52).

The following theorem holds: if conditions (10), (30), (42), (46), (50) and (53) are satisfied, then the solution to problem 1 exists and is unique.

Thus, the existence and uniqueness of the solution to problem 1, a fourth-order equation with two lines of change of type within the considered domain, has been proven. The following works are devoted to a systematic study of various formulations of boundary value problems for third-, fourth- and higher-order equations in domains with certain geometric configurations.

In the work of A.G. Khodjaniyazov [14], the boundary value problem for a fourth-order equation with a spectral parameter, the elliptic part of which is of the fourth order, is investigated. The author managed to find conditions for the spectral parameter that guarantee both the existence and uniqueness of the solution to the problem under consideration. Y.P. Apakov & S.M. Mamajonov [15] considered the boundary value problem for a fourth-order parabolic-hyperbolic equation in a pentagonal domain with two characteristic lines of change of type. The existence and uniqueness of the solution are proved. It is noted that the sought function satisfies a number of boundary conditions and gluing conditions.

In the study by A.A. Klyachin & I.Yu. Verevkin [16], one approach to constructing continuously differentiable piecewise quadratic functions on a triangular mesh is presented, based on smoothing a piecewise linear function in the vicinity of the edges and nodes of the triangulation. The developed method does not require solving systems of linear algebraic equations as in the construction of splines. This circumstance allows this class of functions to be applied for the approximate solution of boundary value problems of 4^{th} order equations.

The author V.V. Karachik [17] presented a representation of the solution to the Dirichlet problem for a homogeneous polyharmonic equation in a unit ball through the solutions to the Dirichlet problems for the Laplace equation. It should be noted that for a composite equation of high order, the elliptic part may be a polyharmonic equation.

In their work, scientists A.K. Urinov & M.S. Azizov [18] formulated and investigated an initial-boundary value problem for a high-order even equation that degenerates at the boundary of the domain. Using the Green's function method and Fourier series theory, they proved the existence, uniqueness, and stability of the solution to the problem under investigation.

The article by A.K. Urinov & D.A. Usmonov [19] is devoted to the study of a non-local initial-boundary value problem for a single fourth-order mixed-type equation in a rectangular domain. The method of separation of variables was applied, and a spectral problem for an ordinary differential equation was obtained. The Green's function of the latter problem is constructed, which reduces it to a Fredholm integral equation of the second kind with a symmetric kernel, from which the existence of eigenvalues and a system of eigenfunctions of the spectral problem follows. An estimate for the solution of the problem is obtained, from which its continuous dependence on the given functions follows.

Zh.A. Balkizov [20] investigated three local boundary value problems for a third-order hyperbolic model equation, the solutions of which are written in explicit form. Using methods of mixed-type equation theory, the existence and uniqueness of the corresponding problems are proved. The formulas found for representing the solutions to the problems can be applied when solving various problems similar to those studied in the work.

The author K.S. Goziev [21] focused on proving the existence and uniqueness of the solution to the boundary value problem for a fourth-order mixed-type equation considered in a limited domain of the plane. To establish the uniqueness of the solution, the method of energy integrals was applied – a classical approach that allows one to evaluate the behaviour of solutions and eliminate ambiguity. The proof of the existence of the solution was reduced to an equivalent formulation in the form of a Fredholm integral equation of the second kind. This transition allows the use of powerful tools from functional analysis and integral equation theory to study the problem at hand. The results obtained expand the understanding of boundary value problems for high-order equations with composite structure and open up prospects for their practical application in various areas of mathematical physics.

A.B. Bekiev & E.E. Eshmuratov [22] conducted a study of the initial-boundary value problem for a degenerate fourth-order equation considered in a rectangular domain. The work focused on constructing a solution in the form of a Bessel function expansion, which allowed the solution to be presented in analytical form. The authors analysed in detail the dependence of the convergence of the obtained series on the initial functions, identifying the conditions under which the solution is guaranteed to converge to the desired function. In addition, the uniqueness of the solution is proven based on its representation as a series, as well as thanks to the completeness properties of the system of well-defined functions used in the methodology. This work expands the class of equations for which analytical solutions can be found and confirms the applicability of functional analysis methods and special functions to complex problems in mathematical physics.

In the work of O.Kh. Abdullaev & A.A. Matchanova [23], boundary value problems for a mixed third-order parabolic-hyperbolic differential equation with a fractional Gerasimov-Caputo operator were studied. The necessary classes of given functions ensuring the unique solvability of the posed boundary value problems were determined. The existence and uniqueness of the solution to the boundary value problem were proved.

N. Mironov [24] presented the formulation of Darboux's problem and the definition of the Riemann-Hadamard function for a third-order equation with a dominating partial derivative (Bianchi's equation). Based on the possibility of representing the Riemann function in an explicit form for a class of equations equivalent to the third-order Bianchi equation, sufficient conditions for the coefficients of the Bianchi equation were proposed. These conditions ensure the construction of the Riemann-Hadamard function in terms of hypergeometric functions.

Conclusions

The article proves the existence and uniqueness of problem 1 for fourth-order composite and hyperbolic equations in a domain with two lines of type change. Using methods from the theory of mixed-composite equations, the main problem 1 is reduced to three independent auxiliary problems. At the same time, according to the formulation of problem 1, as a consequence, on the line of change of equation types, conjugation conditions arise in which the values of the sought function and its derivatives are specified. Using the method of order reduction, Green's and Riemann functions, as well as integral equations, the auxiliary problems in the corresponding subdomains of the considered domain are solved.

Particular attention was paid to the solution of problem 1 to the equations obtained on the line of change of equation types. These equations are expressed in the form of ordinary second-order differential equations with boundary conditions. In addition, problems for hyperbolic equations with given conditions are equivalently reduced to integral equations of the Volterra and Fredholm types of the second kind.

The results of the work can be generalised to the case of similar high-order equations with corresponding boundary conditions and the condition of gluing on the line of change of equation types, as well as in areas with curved boundaries. The course of the research and the results obtained can be used to develop the theory of boundary value problems for non-classical equations of mathematical physics, including mixed and mixed-composite equations, as well as when the line of change of types is not a characteristic.

In the future, it is necessary to study the problem with a normal derivative in that part of the domain where the equation is of the corresponding type. This problem can be equivalently reduced to a singular integral equation and a formula for calculating the problem index can be derived. In addition, when solving the aforementioned problems, sufficient conditions for unique solvability can be derived in explicit form, i.e., conditions ensuring the correctness of the problem under study.

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Conflict of Interest

None.

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Төртүнчү даражадагы курамдуу жана гиперболикалык типтеги теңдеме үчүн эки түр өзгөрүү сызыгы менен сопряжение маселеси

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Аннотация. Жогорку даражадагы аралаш жана аралаш-курамдуу типтеги теңдемелер үчүн чек шарттуу маселелер жылуулуктун, толкундардын таралышы жана аз илешкек чөйрөлөрдүн кыймылы менен байланышкан кубулуштарды математикалык моделдөөнүн маанилүү аспектисин түзөт. Изилдөөнүн актуалдуулугу теңдеменин түрү өзгөргөн сызыктар жана татаал чек шарттар болгон шарттарда мындай маселелерди так анализдөөнүн зарылчылыгы менен негизделет. Иштин максаты – теңдеменин түзүлүшү ар түрдүү болгон үч подобласка бөлүнгөн областта төртүнчү даражадагы курамдуу жана гиперболикалык типтеги теңдеме үчүн чек шарттуу маселени формулировкалоо жана ар тараптуу изилдөө болду. Маселе тиешелүү подобластардагы үч кошумча подмаселеге келтирилип, бул учурда теңдеменин түрү өзгөргөн сызыктарда экинчи даражага чейинки туундулар аркылуу берилген издөөгө тийиш болгон функция үчүн сопряжение шарттары киргизилди. Изилдөөдө чек шарттуу маселелер теориясынын классикалык ыкмалары, теңдеменин даражасын төмөндөтүү ыкмасы, ошондой эле аралаш-курамдуу типтеги теңдемелер теориясынын ыкмалары колдонулган. Ар бир кошумча маселе стандарттык формаларга – Дирихле, Гурса жана Дарбу маселелерине келтирилген. Түрү өзгөргөн сызыктарда экинчи даражадагы дифференциалдык теңдемелер алынган жана алар үчүн так курулган Грин функциялары аркылуу чек шарттуу маселелер чечилген. Гиперболикалык подмаселелер Вольтерра жана Фредгольм интегралдык теңдемелеринин экинчи түрүнө редукцияланып, алардын өзгөчө чечилүү шарты катары ядролорду баалоо негизинде жетиштүү шарттар алынган. Натыйжада ар бир подобласт үчүн чечимдин так аналитикалык формулалары алынды. Алынган жыйынтыктар гетерогендик чөйрөлөрдөгү процесстерди талдоодо жана математикалык физика маселелеринде сандык моделдерди курууда колдонулушу мүмкүн

Негизги сөздөр: чек шарттуу маселелер; Дирихле маселеси; Дарбу типтеги маселе; Грин функциясы; сопряжение шарттары; чек шарттар; Вольтерра жана Фредгольм тендемелери

Задача сопряжения для уравнения составного и гиперболического типов четвертого порядка с двумя линиями изменения типа

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Аннотация. Краевые задачи для уравнений смешанного и смешанно-составного типов высокого порядка играют важную роль в математическом моделировании явлений, связанных с распространением тепла, волн и движением слабовязких сред. Актуальность исследования обусловлена необходимостью строгого анализа таких задач, особенно в условиях наличия линий смены типа уравнения и сложных граничных условий. Целью работы было формулирование и всестороннее исследование краевой задачи для уравнения четвёртого порядка составного и гиперболического типов в области, разделённой на три подобласти с различной структурой уравнений. Задача была приведена к трём вспомогательным подзадачам в соответствующих подобластях, при этом на линиях изменения типа уравнений вводились условия сопряжения, выраженные через искомую функцию и её производные до второго порядка. Использовались классические методы теории краевых задач, приём понижения порядка уравнений, а также методы теории уравнений смешанно-составного типа. Каждая вспомогательная задача была сведена к стандартным постановкам – задачам Дирихле, Гурса и Дарбу. На линиях смены типа получены дифференциальные уравнения второго порядка, для которых решены краевые задачи с использованием явно построенных функций Грина. Гиперболические подзадачи редуцированы к интегральным уравнениям Вольтерра и Фредгольма второго рода, и получены достаточные условия их однозначной разрешимости через оценки ядер. В результате получены явные аналитические выражения решений в каждой подобласти. Результаты могут быть применены для анализа процессов в неоднородных средах и при построении численных моделей в задачах математической физики

Ключевые слова: краевые задачи; задача Дирихле; задача типа Дарбу; функция Грина; условия согласования; краевые условия; уравнения Вольтерра и Фредгольма