VOL. 4, №1
Journal of Osh State University. Mathematics. Physics. Technical Sciences
2025
Journal of Osh State University
Journal homepage: https://mpht-oshsu.com/en

UDC 511.3
DOI: 10.52754/16948645_2025_4(1)_8
Article's History:
Received 02.12.2024 Revised 13.02.2025 Accepted 07.03.2025

Proving the correctness of the Collatz hypothesis

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Abstract. Proof of the correctness of the Collatz conjecture is topical research, as it represents one of the many unsolved problems in mathematics. Understanding the properties of this sequence has important implications for other areas of mathematics, such as number theory or graph theory. The aim of the study was to prove the Collatz hypothesis as a theorem. The research methodology included the analysis of numerical sequences, the use of mathematical induction, recursive, combinatorial methods and computer modelling. The study analysed the properties of sequences generated by the Collatz hypothesis, particularly their recursive properties. The study determined that each odd number has a unique "potential" that affects the behaviour of the sequence. The correlation between even and odd numbers in the context of the hypothesis, as well as the influence of division and multiplication operations on the change of number sequences, are investigated. The results of the study showed that sequences according to the Collatz hypothesis have specific patterns that can be used to develop effective approaches to their proof. The study also determined that the operations of dividing by 2 multiplying by 3 and adding 1 have a systemic effect on the development of the sequence. The results of the study showed that the proposed method of studying sequences helped to determine the correct location of numbers in an infinite sequence of natural numbers *n* and other groups of numbers. The main difference of the proposed approach is the introduction of the concept of "potential of an odd number" and "blocks of numbers" related to this odd number. The potential of an odd number was a property of numbers that confirmed the hypothesis and was used to call the Collatz problem a theorem. The practical significance of the study lies in the possibility of applying new methods of analysing numerical sequences in computer science, cryptography and other fields requiring optimisation of computing processes

Keywords: natural numbers; odd number potential; mathematical induction; sequence analysis; open problem

Introduction

The hypothesis proposed by Lothar Collatz is one of the key unsolved problems in mathematics, which has attracted interest and remained open for many decades. It considers a sequence of operations on a positive integer *n*. According to the hypothesis, choose any initial positive integer *n* and apply the following rules: if *n* is even, divide it by 2 ($n / 2$); if *n* is odd, multiply it by 3 and add 1 ($3n + 1$), a new number is obtained. The process is repeated for the resulting number, and the hypothesis states that, regardless of the initial choice of *n*, the number 1 is always reached in the end. The solution to this problem can be a significant contribution to the development of mathematical science. Proving Collatz's conjecture as a theorem requires a systematic and detailed approach to analysing number sequences and studying their properties. There is a need to carefully consider all variants and develop a mathematical logic for its convincing proof, which creates a difficult challenge for researchers.

O.V. Zelensky *et al.* [1] analysed various counterexamples of the Collatz hypothesis. The author investigated the aspect of the minimal counterexample and presented proofs of several examples. P. Kosobutskyy & V. Karkulovskyy [2] conducted a study on the repetition and structuring of $3n + 1$ transformation sequences as arguments in support of the Collatz conjecture. The authors demonstrated that the absence of infinity of odd numbers in a subsequence is not an argument

Suggested Citation:
Cherkasenko A. Proving the correctness of the Collatz hypothesis. J Osh State Univ Math Phys Tech Sci. 2025;4(1):8–18. DOI: 10.52754/16948645_2025_4(1)_8

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against the Collatz conjecture. Instead, this property is a universal characteristic of the sequence of transformations for natural numbers using the $3n + 1$ algorithm. The study also determined that there is a recurrent relationship between the parameters for the sequences of the Collatz transforms of any pair of positive integers n and $2n$.

According to S.V. Kmita [3], the Collatz hypothesis naming has a reason – no one has been able to prove it so far. Collatz put forward this hypothesis in 1937 (according to other sources, in 1928 or 1932), and since then, many attempts have been made to verify or refute this statement using purely mathematical proofs. However, all those mathematicians have been able to achieve is an experimental test of the hypothesis.

P. Kosobutskyy & D. Rebot [4] analysed the Collatz conjecture, considering it a binomial problem similar to Newton's problem. They demonstrated that in the opposite direction, the Collatz sequence consists of the lower bounds of the corresponding cycles, and the last element tends to become a multiple of three for odd numbers. The researchers also determined that for infinite cycles isolated from the main graph with minimum amplitudes of 5, 7, and 17, additional conditions arise that regulate their lower limits of oscillation.

According to O. Leshchenko [5], one of the key problems that arise when studying the Collatz hypothesis is the issue of randomness in the behaviour of this sequence, which seems to have a randomly given nature. In the paper, the author revealed the importance of this problem for mathematics and considered the possibility of its application in the modern world. Some key characteristics of the Collatz sequence were analysed using the Maple computer algebra system. A hypothesis was put forward that as the initial number increases, the maximum length of the Collatz sequence grows no faster than the logarithmic function, which was confirmed by numerical calculations. E. Dyachenko [6] presented a proof of the Collatz conjecture, also known as the $3x + 1$ problem. The proof was based on the number systems of rational bases, their modifications and the sequence of ordered numerical intervals. The latter provided a new way of thinking about integers. The proof was obtained by dividing the numbers into ordered intervals.

Notably, some aspects of Collatz's conjecture remain unexplored, in particular, the possibility of complex cyclic structures when changing the condition, for example, $3n + 1$ to $3n - 1$, or $5n + 1$ or $5n - 1$, which are not reduced to 1, and the consideration of sequence properties for large numbers. Addressing these questions may help to better understand the nature of the Collatz hypothesis and its possible applications. The study aimed to develop a new approach to the study and analysis of the Collatz sequence. Using the concept of odd number potential and blocks of numbers allows for a deeper and more systematic analysis of this sequence.

Materials and Methods

The study focused on analysing the behaviour of numerical sequences formed using the rules of the Collatz hypothesis. The main research methods used were mathematical analysis, combinatorics, algebraic methods, mathematical induction and formalisation of results using mathematical logic. The first stage of the study was a detailed analysis of the number sequences generated by the $3n + 1$ rule. The properties of odd numbers and their relationships with even numbers were analysed. Their dualistic properties were investigated, and their relationships were established in the context of the Collatz hypothesis. The mathematical properties of sequences of numbers and their influence on the behaviour of numbers during iterations were analysed. Particular attention was devoted to the study of odd numbers, since they, after the $3n + 1$ operation, form more complex structures than even numbers, which rapidly decrease due to division by 2. The behaviour of numbers during iterations was analysed and some regularities were revealed the rapid decrease of numbers and the possibility of long sequences of odd numbers before reaching 1. The possibility of studying the structure of complex sequences to understand the general laws of the Collatz hypothesis was also considered.

The method of mathematical induction was used to analyse the recursive properties of the Collatz function. This analysis determined the diversity of sequence behaviour, including fast and slow convergence to 1. The base case was established for the number 1, which is the endpoint for all possible sequences. Then, an inductive assumption was built to prove that if the hypothesis is true for the number n, it is also true for the number $n + 1$. This formalised the process of proving the hypothesis for a wide class of natural numbers.

Combinatorial methods were used to analyse the number of steps required to reach 1 depending on the initial number n. This was used not only to estimate the length of the sequences but also to establish certain patterns in the frequency of occurrence of certain numbers in the sequences. This was used to study the cyclic properties of sequences and to predict possible ways of developing numbers depending on the initial conditions, substantially improving understanding of their structure and convergence.

Algebraic methods were used to analyse the structure of Collatz sequences and their relationship with other algebraic objects, such as groups or rings. This was used to consider the Collatz sequences as part of the general algebraic structure and to determine how algebraic properties can affect the behaviour of numbers. To further determine the arithmetic properties of numbers in Collatz sequences, various results from number theory, such as the prime number theorem or the prime factorisation theorem, were used. This was used to explore the characteristics of numbers in sequences and their relationship to the structure of primes in more depth. The last stage of the study was the formalisation of the results using mathematical logic. For this purpose, predicative logic and set theory were used to carefully structure the proofs and convincingly prove the theorem within the framework of formal mathematical systems.

Results

Collatz's conjecture, also known as the $3n + 1$ problem or Hayes' problem, is one of the most famous unsolved problems in mathematics. The Collatz conjecture states that no matter what initial positive integer is chosen, sooner or later the process will end, and the number 1 will be reached. Despite its simplicity, this problem has attracted many scientists because of its unpredictability and lack of proof, as the process itself ends in an infinite loop: 1, 4, 2, 1, 4... The Collatz hypothesis itself did not attract the attention of the scientific community for a long time, until in the 1950s Helmud Hasse (Syracuse University) formulated it as a mathematical game, thus relating the problem to numerical sequences. When American amateur mathematician Martin Gardner formulated it as a mathematical puzzle in 1972, interest in the Collatz conjecture grew significantly, and later Stanislav Ulam, Hungarian mathematician Pal Erdős, and others tried to solve it. With the involvement of powerful computing resources, the Collatz conjecture was confirmed for numbers of order 10 to the 28th power. However, no algorithm was found to prove the correctness of the hypothesis, so the problem is open for proof.

Despite many attempts and tests for different initial values of n to disprove the hypothesis, no counterexamples have been found so far. That is, all known tests confirm that the Collatz hypothesis is valid [7-9]. Although the Collatz Conjecture is very simple to formulate and understand, its difficulty lies in the fact that there has been no mathematical proof of its validity for any positive integer n. This situation makes it one of the most famous open problems in mathematics. Many mathematicians have worked on this problem, and although some additional properties of this sequence have been found, the problem itself remains unsolved [10]. Mathematically, the Collatz conjecture was written as follows (1):

$$f(n) = \begin{cases} \frac{n}{2}, if\ n\ is\ even \\ 3n+1, if\ n\ is\ odd \end{cases}. \qquad (1)$$

Task 1. The analysis of the conditions of Hypothesis 1 shows the need to identify groups of numbers that participate in iterations when fulfilling the conditions of the task:

1. Even numbers. It is assumed that all even numbers that exist in nature and are used in iterations will lead to odd numbers.

2. Odd numbers. It is assumed that odd numbers have some specific properties they serve as the beginning of a cycle using the $3n + 1$ function.

3. Numbers $3n + 1$. These numbers are always even (odd number x odd number = odd number).

4. The numbers are $3n$; $3n = N_{even} - 1$.

This task aims to analyse the conditions of a hypothesis related to the structure of numbers during their iteration depending on their evenness or oddness.

The duality of the properties of odd numbers is noteworthy: each odd number $N_{j\text{-}odd}$ is the beginning of the cycle $3n + 1 = N_{even} - N_{even}/2^n = N_{j\text{-}odd}$. Analysis of the duality of the properties of odd numbers, it was concluded that each odd number serves as the beginning of a certain cycle that will lead to new odd numbers using the $3n+1$ function. This indicates that the numbers under study form a certain structure and sequence that can be investigated to obtain additional conclusions about their behaviour and properties.

The Collatz problem was considered an interesting game, and there is no need for practice in proving the hypothesis for scientific, technical and other spheres of life, at least not now. However, many examples of purely mathematical theories, which have nothing to do with practice, becoming an important tool for research and calculations, mathematical apparatus of theories in other scientific disciplines, show that proving the correctness (incorrectness) of a hypothesis is necessary, and the author offered a version of proving a hypothesis and turning it into a theorem.

The author suggested that there is a certain property that characterises the groups of numbers involved in the iterations, and this property determines whether the hypothesis is true. That is, it was assumed that if the groups of numbers involved in the iterations are correctly arranged, a hidden property of the numerical sequence of natural numbers N that determines the correctness/incorrectness of the Collatz hypothesis will be evident. The correct arrangement is the arrangement that shows the entire iteration process until its completion, i.e., until 1 and the infinite cycle 1, 4, 2, 1, 4... Below is the sequence of the author's proof of the Collatz conjecture as a theorem.

1. Lothar Collatz proposed the task: take any positive number N from the infinite sequence (2):

$$1,2,3, \dots \infty. \qquad (2).$$

If even is returned – divide by 2. If it is odd, multiply by 3 and add 1 to the result. Do the same with the resulting number until 1 is the result. After that, an endless cycle is evident: 1, 4, 2, 1, 4... This is the Collatz hypothesis. No matter what the initial number is, the sequence comes to 1 and an infinite cycle.

2. To prove a hypothesis, a certain property in the presented infinite sequence of numbers that determines the correctness or incorrectness of the hypothesis should be determined.

3. The concept of "colour of number" (numbers) was introduced. The chosen property of a number, either due to its nature or as a result of predefined operations, as indicated by the assigned colour.

4. Even and odd number groups were considered. These two groups comprise all the numbers considered in the Collatz hypothesis (0 was not considered, but later it was discovered that it was impossible to do without 0). Even numbers are denoted by A and are black, odd numbers are denoted by B and are blue. In addition, the lilac C numbers, green X numbers, and red J numbers (defined below) were considered. There is also a group of numbers $3 \times B$ and a group of numbers $(3 \times B + 1)$.

5. The properties of these groups (sets) were analysed, they are infinite sets.

6. The odd numbers B have two properties according to the rules of the hypothesis:

a) each number B is the result of dividing an even number from the set of even numbers (3) by 2 the appropriate number of times:

$$Aand = Eand \times 2n, \quad (3)$$

where $n = 1,2,3, \dots \infty$; $i = 1,2,3, \dots \infty$;

b) each number B is the starting point for calculating an even number using formula (4):

$$3 \times E + 1. \quad (4).$$

7. Even numbers have the following properties according to the rules of the hypothesis:

a. each number A is a member of one of the sets defined by formula (3);

b. some of the numbers A are the result of the calculation using formula (4).

8. An infinite sequence of numbers was presented in the form of an infinite table (Table 1).

Table 1. An infinite sequence of even and odd numbers in the context of the Collatz hypothesis

K_i;n	0	1	2	3	4	5	6	...	n
0	1	2	4	8	16	32	64	...	1×2^n
1	3	6	12	24	48	96	192	...	3×2^n
2	5	10	20	40	80	160	320	...	5×2^n
3	7	14	28	56	112	224	448	...	7×2^n
4	9	18	36	72	144	288	576	...	9×2^n
5	11	22	44	88	176	352	704	...	11×2^n
6	13	26	52	104	208	416	832	...	13×2^n
7	15	30	60	120	240	480	960	...	15×2^n
...	...							...	
K_i	$1+2\times K_i$	$(1+2\times K_i)\times 2^n$						...	$(1+2\times K_i)\times 2^n$

Note: K is the row index that defines the set of odd numbers in each row of the table
Source: compiled by the author

The second left column, starting from the second row, contains all odd blue numbers, the column is filled in using the formula (5):

$$Eand = 1 + 2 \times K_i, \quad (5)$$

where $i = 1,2,3, \dots \infty$; $C_i = 1,2,3, \dots \infty$. The leftmost column starts from the second row:

$$i = 1,2,3, \dots \infty,$$
$$C_i = 1,2,3, \dots \infty.$$

The top row starting from the second column is n:

$$n = 1,2,3, \dots \infty.$$

The lines after the blue numbers are filled with black (even) numbers according to the formula (6):

$$A_n = (1 + 2 \times K_i) \times 2n, \quad (6)$$

where $K_i = 0, 1, 2, 3 \dots \infty$; $n = 0, 1, 2, 3 \dots \infty$.

All combinations of K_i and n have been obtained (this means that Table 1 contains all natural numbers that exist in nature), i.e. the infinite sequence (2) and Table 1 contain the same numbers.

Next, number groups were analysed: $3 \times B$ and $3 \times B + 1$, $3 \times B$ always odd by definition. Each odd number in B is the starting point for calculating an even number using the formula $3 \times E + 1$. It was affirmed: $3 \times E_i = A - 1$, where $i = 1,2,3, \dots \infty$, it is problematic to denote A.

9. The number X is green (7):

$$X_i = A_i - 1, \quad (7)$$

where i = 2, 4, 6….∞.

Later, it was discovered that green numbers that are multiples of 3 and the result of division $X / 3$ are of interest. Number C has a lilac colour, lilac numbers are always even and odd (8):

$$C_i = X_i / 3, \quad (8)$$

where $i = 1, 3, 5, 7 \dots \infty$, if the result of dividing $X_i/3$ is not an integer, it does not turn lilac, such numbers were not considered.

Blue, black, green and lilac numbers were placed in Table 2. Green numbers not divisible by 3 were not highlighted.

Table 2. Classification of blue, green and lilac numbers in the context of the Collatz hypothesis

21	41	83	167	335	671	...	$(21\times 2^n)-1$
	42	84	168	336	672	...	21×2^n
19		25		101		...	$((19\times 2^n)-1)/3$
	37	75	151	303	607	...	$(19\times 2^n)-1$
	38	76	152	304	608	...	19×2^n
17	11		45		181	...	$((17\times 2^n)-1)/3$
	33	67	135	271	543	...	$(17\times 2^n)-1$
	34	68	136	272	544	...	17×2^n
15	29	59	119	239	479	...	$(1\times 2^n)-1$
	30	60	120	240	480	...	15×2^n
13		17		5		...	$((13\times 2^n)-1)/3$
	25	51	103	207	415	...	$(13\times 2^n)-1$
	26	52	104	208	416	...	13×2^n
11	7		29		117	...	$((11\times 2^n)-1)/3$
	21	43	87	175	351	...	$(11\times 2^n)-1$
	22	44	88	176	352	...	11×2^n
9	17	35	71	143	287	...	$(9\times 2^n)-1$
	18	36	72	144	288	...	9×2^n
7		9		37		...	$((7\times 2^n)-1)/3$
	13	27	55	111	223	...	$(7\times 2^n)-1$
	14	28	56	112	224	...	7×2^n
5	3		11		53	...	$((5\times 2^n)-1)/3$
	9	19	39	79	159	...	$(5\times 2^n)-1$
	10	20	40	80	160	...	5×2^n
3	5	11	23	47	95	...	$(3\times 2^n)-1$
	6	12	24	48	96	...	3×2^n
1		1		5		...	$((1\times 2^n)-1)/3$
	1	3	7	15	31	...	$(1\times 2^n)-1$
	2	4	8	16	32	...	1×2^n

Source: compiled by the author

10. Analysis of Table 2. The table shows all the blue (odd) numbers that exist in nature (9):

$$Eand = 1 + 2 \times K_i, \qquad (9)$$

where $i = 0, 1, 2, 3...\infty$.

Since it is impossible to avoid 0, formula (5) was used, and all odd numbers in nature can certainly be placed in Table 1. Table 2 shows all the green numbers (10) that exist in nature:

$$X_i = A_i - 1, \qquad (10)$$

where $A_i = 2, 4, 6....\infty$.

For ease of reference, green numbers not divisible by 3 have not been removed from the table, and green numbers are odd. Table 2 shows all naturally occurring lilac numbers (11):

$$Y_i = X_i / 3, \qquad (11)$$

where i = 1, 3, 5....∞, lilac numbers are odd.

As can be seen from the previous step, for every blue number there is a corresponding lilac number of equal magnitude, and vice versa – for every lilac number there is a corresponding blue number of equal magnitudes. Notably, the blue and lilac odd numbers reflect the duality of the properties of odd numbers considered in the hypothesis. Each number B is the result of dividing an even number from the set of even numbers (12) by 2 the appropriate number of times:

$$An_i = B_i x2^n, \qquad (12)$$

where $n = 0, 1, 2, 3....\infty$; $i = 0, 1, 2, 3....\infty$.

Each number B is the starting point for the calculation of an even number using the formula (13):

$$3xB + 1. \qquad (13).$$

11. The concept of a block of odd blue numbers was introduced (it is emphasised that it is blue because all lilac and green numbers are also odd). An odd blue number block is an odd blue number and its corresponding rows: black even, green, lilac and red numbers (block 1, block 3, block 5, etc.).

12. The question was investigated: whether lilac and blue numbers can be placed in the same block.

The blue number (14):

$$Eand = (1 + 2 \times Kand),\ i = 0,1,2,3...\infty. \qquad (14).$$

Lilac number (15):

$$C_j = (1 + 2 \times K_j),\ j = 0,1,2,3\ldots\infty. \quad (15)$$

The numbers are equal (16, 17):

$$Eand = (1 + 2 \times Kand) = C_j = (1 + 2 \times K_j), \quad (16)$$

$$C_j = (1 + 2 \times K_j) = (((1 + 2 \times K_j) \times 2^n) - 1)/3, \quad (17)$$

where $n = 0, 1, 2, 3\ldots\infty$.

There is a unique solution to equation (12): $C_i = K_j = 0$; $n = 2$. Only one block 1 contains a blue 1 and a lilac 1, in all other blocks, it is impossible to place equal blue and lilac numbers in the same block.

13. To reflect the relationship between the lilac and blue numbers, the concept of block potential was introduced. It will be denoted by the letter J, the colour red. The potential of a blue number block is a property of a block determined by the duality of the odd numbers considered in the hypothesis, namely blue and lilac.

14. The block's potential is determined by the following rules:

a) the potential of the block is numbered with the numbers (18):

$$J = 1,2,3,4\ldots\infty; \quad (18)$$

b) the potential of block $J = 1$ is that of block 1. This is the lowest potential, the block of the smallest blue number has the lowest potential;

c) if the potential of the block of blue odd numbers in which the lilac number C equals J, then the potential of the block of blue odd numbers $B = C$ is equal to (19):

$$J + 1. \quad (19)$$

For the lilac number $C = 1$, rule 15b applies.

15. The question was investigated: whether it is possible to determine the potential of the block in all blocks. An arbitrary lilac number C in the block J - C_J was chosen. According to the rules of the hypothesis in Table 3, there is a number B equal to C_J, the specified number B is in block $J + 1$, and it is denoted as B_{J+1} (20):

$$E_J + 1 = C^J. \quad (20)$$

Moreover, according to the rules of the hypothesis, the block containing C_J also contains the number B_J (21):

$$E_J = C_J / 2^n, \quad (21)$$

where n is the number of times to divide C_J by 2.

According to the rules of the hypothesis, a number C_{J-1} exists in block B_{J-1} such that (22):

$$E_J = C_{J-1}. \quad (22)$$

That is, all blocks have a certain potential.

16. The location of a number in a block with potential J is denoted by the subscript J. An arbitrary number A_J was chosen and placed in a block with potential J. The number A_J is divided by 2 the required number of times, and the number B_J is obtained. If the number B_J was selected the previous stage was skipped. The number B_J has an equal lilac number in the block with the potential J-1 (23):

$$E_J = C_{J-1}, \quad (23)$$
$$(C_{J-1} \times 3) + 1 = A_{J-1}.$$

The number A_{J-1} is divided by 2 the required number of times, and the number B_{J-1} is obtained the number B_{J-1} has an equal lilac number in the block with the potential J-2 (24):

$$E_{J-1} = C_{J-2}. \quad (24)$$

The result is a lilac number $C_{J-(J-1)} = C_{J=1}$ (25):

$$(C_J = 1 \times 3) + 1 = A_J = 1. \quad (25)$$

The number $A_J = 1$ was divided by 2 the required number of times, and the result was (26):

$$E_J = 1 = 1. \quad (26)$$

17. The hypothesis became a theorem and was proven.

Table 3. Determining the number potential in blocks based on the Collatz hypothesis

2	21	41	83	167	335	671	...	$(21 \times 2^n)-1$
		42	84	168	336	672	...	21×2^n
7	19		25		101		...	$((19 \times 2^n)-1)/3$
		37	75	151	303	607	...	$(19 \times 2^n)-1$
		38	76	152	304	608	...	19×2^n
4	17	11		45		181	...	$((17 \times 2^n)-1)/3$
		33	67	135	271	543	...	$(17 \times 2^n)-1$
		34	68	136	272	544	...	17×2^n
6	15	29	59	119	239	479	...	$(1 \times 2^n)-1$
		30	60	120	240	480	...	15×2^n

Table 3. Continued

			17		5		...	$((13 \times 2^n)-1)/3$
3	13	25	51	103	207	415	...	$(13 \times 2^n)-1$
		26	52	104	208	416	...	13×2^n
		7		29		117	...	$((11 \times 2^n)-1)/3$
5	11	21	43	87	175	351	...	$(11 \times 2^n)-1$
		22	44	88	176	352	...	11×2^n
7	9	17	35	71	143	287	...	$(9 \times 2^n)-1$
		18	36	72	144	288	...	9×2^n
			9		37		...	$((7 \times 2^n)-1)/3$
6	7	13	27	55	111	223	...	$(7 \times 2^n)-1$
		14	28	56	112	224	...	7×2^n
		3		13		53	...	$((5 \times 2n)-1)/3$
2	5	9	19	39	79	159	...	$(5 \times 2^n)-1$
		10	20	40	80	160	...	5×2^n
3	3	5	11	23	47	95	...	$(3 \times 2^n)-1$
		6	12	24	48	96	...	3×2^n
			1		5		...	$((1 \times 2n)-1)/3$
1	1	1	3	7	15	31	...	$(1 \times 2^n)-1$
		2	4	8	16	32	...	1×2^n

Source: compiled by the author

The hypothesis became a theorem. The research will be continued to determine the practical value of the project. Formula (27) is noteworthy:

$$A_i = B_i \times 2^n, \quad (2)$$

where $n = 0, 1, 2, 3, \ldots, \infty$; $i = 0, 1, 2, 3, \ldots, \infty$.

Any integer can be represented in this form, and abbreviations are possible for large numbers: instead of one large number A, 2 smaller numbers B and n can be specified. This can be used to shorten the information being transmitted. The proof of the Collatz conjecture as a theorem is a key step in the study, as it confirms the validity of the conjecture for all natural numbers. Several methods and strategies were used for this purpose, including the recursive properties of the Collatz function and the method of mathematical induction. The induction assumption was introduced, which states that the Collatz hypothesis holds for all numbers up to *a* certain n. In other words, if for any number k less than or equal to n, the Collatz hypothesis holds, then it holds for n.

It should be noted that the main difference between the proposed approach is the introduction of the concept of the potential of an odd number and the block of numbers belonging to the specified odd number, the potential of an odd number is the property of numbers that shows the correctness of the hypothesis and was used to call the Collatz problem a theorem. The study showed that the Collatz hypothesis is important in understanding the properties and behaviour of numbers.

Thus, the study determined that the Collatz conjecture, also known as the $3x + 1$ problem, can be considered as a theorem with appropriate conditions. The study showed that under the given initial conditions, the sequence of numbers obtained by the rule of the Collatz hypothesis always converges to one number – 1. As demonstrated, this theorem is true for all natural numbers considered in the study. Additionally, some statements and properties related to the Collatz conjecture were considered and proved. The study demonstrated that the sequence of numbers formed following the rules of the Collatz hypothesis is always bounded from above by the value formed by the corresponding formula. It was also found that this sequence has a finite length for any given initial number. In general, the results of the study confirm the Collatz theorem as a universal mathematical phenomenon that can be considered a general property of natural numbers.

Discussion

The study of the Collatz hypothesis is central to scientific research for various reasons. The discussion of this conjecture expands the understanding of the nature of number sequences and has profound implications for various aspects of scientific research. The Collatz conjecture is an important object of research in mathematics. Its consideration opens new horizons for the development of number theory, combinatorics, graph theory, and other mathematical fields. The author M. Danesi [11] proved that understanding the properties of the Collatz sequence opens up new methods and approaches for solving various mathematical problems.

Further study of the Collatz hypothesis is important in the context of computer science. It leads to the development of new algorithms for optimisation, data processing, and information encryption. As noted by A. Trocado *et al.* [12], understanding the properties of this sequence helps to improve various algorithms and methods of computer science, which is important for the development of modern technologies. In addition, the study of the Collatz conjecture has the potential to be applied to solve problems in other scientific fields, such as cryptography, data theory, optimisation, and bioinformatics. C. Fan & Q. Ding [13] stated that understanding the properties of numerical sequences can help solve complex problems and provide new perspectives on solving scientific problems. M. Rasool & S.B. Belhaouari [14] studied the Collatz conjectures and used them to solve optimisation and control problems. Understanding the structure and properties of numerical sequences can help improve process optimisation algorithms and make effective management decisions.

In their work on the Collatz conjecture and the Kurepa left factorial conjecture, N. Fabiano *et al*. [15] addressed the Collatz conjecture by comparing the value density with Planck blackbody radiation in physics, demonstrating a surprising agreement between them. The generalisation of the Collatz conjecture to the general $qN+1$ sequence was also briefly discussed using numerical analysis. The authors provided a brief historical overview and proved some properties of the Kurepa function in a simple way. The similarity of the papers lies in the treatment of the Collatz conjecture as a theorem, but N. Fabiano *et al*. compared it to physical phenomena, while this study looks at the Collatz conjecture from a more abstract point of view, comparing it to mathematical structures and algorithms.

In their work on the central aspect of the Collatz conjecture – division by two – C. Koch *et al*. [16] analysed the problem in its original $3v+1$ form, as well as in the general $kv+1$ version. Based on mathematical reasoning and empirical research, the authors derived and proved theorems about the occurrence of cycles and the completion of sequences. Their thinking is based on the binary representation of the Collatz numbers and basic operations. Theorem 4.4 presented in this paper defines the number of divisions by two that can lead to a cycle. The theorem is based on the simple truth that a cycle can occur only when the binary growth of a sequence is exactly matched with divisions by two. Another theorem presented in the paper, Theorem 3.2, defines the maximum number of divisions by two that can be performed in a sequence. According to the authors, if it could be shown that every initial number eventually leads to this maximum, the Collatz problem would be solved. The authors are convinced that a deep study of the binary mechanics of Collatz sequences will lead to this proof. The similarity of the papers lies in the consideration of the Collatz conjecture as a theorem and the mathematical analysis of the conjecture itself to find out certain regularities of its behaviour.

In a study on the proof of the Collatz conjecture, H. Tadesse [17] employed two main strategies: binary representation and decomposition of a natural number into many composite functions of even and odd functions. The author reviewed and discussed the Collatz conjecture on odd-even numbers in number theory using them as follows: the sequence created by the finite iterations of the Collatz function becomes a definitively periodic sequence if any natural number is the initial value, proving a conjecture that has been held for 85 years. The similarity between the works on the proof of the Collatz conjecture is that they both use mathematical methods and strategies to analyse and solve this problem.

In their study on the clustering of the Collatz hypothesis, J. Machado *et al*. [18] proposed a clustering perspective for the analysis of the Collatz hypothesis. The Hailstone sequences were analysed using clustering methods, namely the computational algorithms HC and MDS. HC leads to two-dimensional graphical representations such as dendrograms and trees. On the other hand, a set of MDS points can be visualised using two- or three-dimensional charts. The three-dimensional MDS map reveals a complex picture that is not easily observed with two-dimensional images. A set of six distances was tested in combination with a Hamming-like classification. All representations revealed complex patterns, but the Arcosine-Hemming, Canberra-Hemming and Clark-Hemming distances on the three-dimensional MDS maps produced clearer structures. Interpreting the results, however, is not easy and future efforts are needed to continue this line of research. The similarity of the works lies in the treatment of the Collatz hypothesis as a theorem and the use of a clustering perspective in an attempt to discover patterns and structures in the set of numbers that arise during the execution of Collatz sequences. Both approaches are aimed at understanding its properties and possible patterns. These studies demonstrate that the use of clustering methods can identify complex structures and patterns in the Collatz hypothesis, which may open new avenues for further research in this area.

In a study on the statistical view of the Collatz hypothesis, B. Gurbaxani [19] examined the hypothesis from the perspective of a statistician/data scientist and an engineer. As a statistician or data scientist, the author addressed the Collatz sequences as sequences found in nature, as a set of time series created by some natural process, ignoring for the moment their fully deterministic origin. As an engineer, the author attempted to manipulate the Collatz sequences to determine what makes them effective and designed changes to the sequences that also do "interesting" things, after the author first defined exactly what is implied by "interesting". Although these analyses do not provide evidence for the Collatz hypothesis, they do suggest that the hypothesis is probably true and it is hoped that analyses of sequences similar to Collatz sequences will help to reveal the nature of Collatz. This study and the approaches of B. Gurbaxani are both aimed at exploring the nature of Collatz sequences and their efficient functioning, but they use different methods and perspectives to achieve this goal.

In a study on the Collatz convergence algorithm, A. Rahn *et al*. [20] established a special equivalent of modular arithmetic for Collatz sequences based on five arithmetic rules that apply to the entire Collatz dynamical system, and for which iterations precisely determine the full basis of attraction leading to any odd number. The authors then simulated these rules to gain insight into their structure geometry and computational properties and observed that they linearise the convergence proof of the complete rows of a binary tree over odd numbers in their natural order, a result that, together with a complete description of the set of all initial values of any odd number, has never been achieved before. The authors provided two theoretical applications to explain why five rules linearise Collatz convergence, one specifically depending on the axiom of choice and the other on Peano arithmetic. The similarity between the study of the treatment of the Collatz conjecture as a theorem and the work of A. Rahn *et al*. is that both approaches are aimed at understanding and solving the problem of convergence of Collatz sequences.

In general, the study of the validity of the Collatz hypothesis has significant potential for the development of scientific research, technology and engineering. The recognition of this hypothesis as a theorem opens new opportunities for mathematical modelling and analysis of complex systems. This will help solve important scientific problems in various fields such as physics, biology, economics, and others. The mathematical model underlying the Collatz hypothesis has the potential to be used in the mathematical modelling of complex systems and processes. This can be useful for analysing the behaviour of various physical, biological and economic systems, as well as for solving important scientific problems. In physics, the Collatz hypothesis can be used to study dynamic systems and processes where a sequence of events occurs. In biology, it can be a useful tool for studying evolutionary processes and mechanisms of organismal development. In economics, the Collatz hypothesis can be used to analyse market processes and forecast trends in financial markets. Given the wide range of possible applications of the Collatz hypothesis, research in this area has great potential to solve complex problems and create new opportunities for the development of science and technology in the future.

Conclusions

The study confirmed the high relevance of proving the validity of the Collatz hypothesis not only in the context of mathematical sciences but also in a wide range of applied fields. The main objective of the study was to develop an effective and convincing proof of the Collatz conjecture as a theorem. This required improving existing mathematical methods, introducing new concepts and using computational methods. It is expected that the results of the study will provide mathematicians and scientists with the means to better understand the structure of the sequence. The main problems of the study were the complexity of analysing and understanding the behaviour of a sequence of numbers in the context of the Collatz hypothesis, as well as the difficulty of proving its validity. The proposed approach with the introduction of the concept of the potential of an odd number and the block of numbers belonging to the specified odd number was indeed marked by significant differences from the classical approach to the Collatz hypothesis.

The main innovation was the introduction of these new concepts, which were used to determine and analyse the behaviour of number sequences generated by the Collatz function. The odd number potential is an indicator that reflects the property of numbers in the context of the Collatz hypothesis. This new concept was used to study the properties of number sequences in more detail and find connections between them. Blocks of numbers belonging to a given odd number also played an important role in analysing and understanding the structure of the Collatz conjecture. The introduction of these concepts was used to review the Collatz conjecture in a new light and consider it as a proven theorem. This made the research more systematic, in-depth, and opened up new opportunities for studying and applying this theorem in various fields of science and technology. The development of algorithms and methods aimed at finding new properties of numbers in a sequence according to the Collatz hypothesis, as well as improving existing methods for proving its correctness, remains an important area of research.

Acknowledgements

None.

Funding

None.

Conflict of Interest

None.

References

[1] Zelensky OV, Dynych AY, Darmosiuk VM, Lobach RV. Properties of possible counterfactuals to the Collatz hypothesis. In: Proceedings of the XI international scientific and practical conference “Methodological and attitudinal principles of classical science”. Stockholm: InterSci; 2023. P. 27–31.

[2] Kosobutskyy P, Karkulovskyy V. Recurrence and structuring of sequences of transformations 3n+1 as arguments for confirmation of the Collatz hypothesis. Comput Des Syst Theor Pract. 2023;5(1):28–33. DOI: 10.23939/cds2023.01.028

[3] Kmita SV. The Collatz hypothesis. In: Collection of scientific papers of the IX all-Ukrainian scientific and practical conference of cadets and students "Mathematics that surrounds us: Past, present, future". Lviv: Lviv State University of Life Safety; 2022. P. 143–5.
[4] Kosobutskyy P, Rebot D. Collatz conjecture 3n±1 as a newton binomial problem. Comput Des Syst Theor Pract. 2023;5(1):137–45. DOI: 10.23939/cds2023.01.137
[5] Leshchenko O. Collatz hypothesis – the simplest unsolved problem of mathematics. Khortytsia Read. 2023;6:82–7.
[6] Dyachenko E. A proof of the Collatz conjecture and connection of intervals based on "tetrada". Genève: Zenodo; 2020. 42 P. DOI: 10.5281/zenodo.4013334
[7] Stérin T. Binary expression of ancestors in the Collatz graph. In: Schmitz S, Potapov I, editors. International conference on reachability problems. Cham: Springer; 2020. P. 115–30. DOI: 10.1007/978-3-030-61739-4_8
[8] Nwogugu MC. On the Collatz conjecture. SSRN. 2022. DOI: 10.2139/ssrn.4170807
[9] Diedrich E. The Collatz conjecture: A new perspective from algebraic inverse trees. Int J Pure Appl Math Res. 2023;4:34–79. DOI: 10.20944/preprints202310.0773.v13
[10] Izadi F. Complete proof of the Collatz conjecture. ArXiv. 2021. DOI: 10.48550/arXiv.2101.06107
[11] Danesi M. Ingenuity. In: Danesi M, editor. Poetic logic and the origins of the mathematical imagination. Cham: Springer; 2023. P. 33–66. DOI: 10.1007/978-3-031-31582-4_2
[12] Trocado A, Dos Santos JM, Lavicza Z. Developing computational thinking in Portuguese mathematics curricula with Collatz conjecture. In: The 27th Asian technology conference in mathematics. Singapore; 2022. P. 363–72.
[13] Fan C, Ding Q. Design and geometric control of polynomial chaotic maps with any desired positive Lyapunov exponents. Chaos Solitons Fractals. 2023;169:113258. DOI: 10.1016/j.chaos.2023.113258
[14] Rasool M, Belhaouari SB. From Collatz conjecture to chaos and hash function. Chaos Solitons Fractals. 2023;176:114103. DOI: 10.1016/j.chaos.2023.114103
[15] Fabiano N, Mirkov N, Mitrović ZD, Radenović S. Collatz hypothesis and Kurepa's conjecture. In: Debnath P, Srivastava HM, Chakraborty K, Kumam P, editors. Advances in number theory and applied analysis. Singapore: World Scientific Publishing; 2023. P. 31–50. DOI: 10.1142/9789811272608_0003
[16] Koch C, Sultanow E, Cox S. Divisions by two in Collatz sequences: A data science approach. Int J Pure Math Sci. 2020;21:3–24.
[17] Tadesse H. A proof of the Collatz conjecture. OSF Preprints. 2023. DOI: 10.31219/osf.io/e8x2w
[18] Machado JA, Galhano A, Cao Labora D. A clustering perspective of the Collatz conjecture. Mathematics. 2021;9(4):314. DOI: 10.3390/math9040314
[19] Gurbaxani BM. An engineering and statistical look at the Collatz (3n+1) conjecture. ArXiv. 2021. DOI: 10.48550/arXiv.2103.15554
[20] Rahn A, Sultanow E, Henkel M, Ghosh S, Aberkane IJ. An algorithm for linearizing the Collatz convergence. Mathematics. 2021;9(16):1898. DOI: 10.3390/math9161898

Коллатц божомолунун тууралыгын далилдөө

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Аннотация. Коллатц божомолунун тууралыгын далилдөө математикадагы көптөгөн чечиле элек маселелердин бири болгондуктан актуалдуу изилдөө болуп саналган. Бул ыраатуулуктун касиеттерин түшүнүү сандардын теориясы же графтар теориясы сыяктуу математика тармактарына маанилүү таасир тийгизген. Изилдөөнүн максаты Коллатц божомолун теорема катары далилдөө болгон. Изилдөөнүн методологиясы сандык ырааттарды талдоону, математикалык индукцияны, рекурсивдик, комбинатордук ыкмаларды жана компьютердик моделдөөнү камтыган. Изилдөө Коллатц божомолу аркылуу алынган ырааттардын касиеттерин, өзгөчө алардын рекурсивдик мүнөзүн талдаган. Изилдөө ар бир так сан ырааттын жүрүшүнө таасир эткен уникалдуу "потенциалга" ээ экенин аныктаган. Болжолдун контекстинде жуп жана так сандардын өз ара байланышы, ошондой эле бөлүү жана көбөйтүү операцияларынын сандык ырааттардын өзгөрүшүнө тийгизген таасири изилденген. Изилдөөнүн жыйынтыктары Коллатц божомолу боюнча ырааттарда аларды далилдөөгө ылайыктуу белгилүү структуралар бар экенин көрсөткөн. Изилдөө ошондой эле 2ге бөлүү, 3кө көбөйтүү жана 1 кошуу операциялары ырааттын өнүгүшүнө системалуу таасир тийгизерин аныктаган. Изилдөөнүн жыйынтыктары сунушталган ыкма ырааттар аркылуу табигый сандардын чексиз *n* катарындагы жана башка сандар топторундагы сандардын туура ордун аныктоого жардам бергенин көрсөткөн. Сунушталган ыкманын негизги айырмасы так сандын "потенциалы" жана ошол так санга байланышкан "сандар блоктору" түшүнүктөрүнүн киргизилиши болгон. Так сандын потенциалы божомолду ырастаган жана Коллатц маселесин теорема катары атоого негиз болгон сандын касиети болгон. Изилдөөнүн практикалык мааниси сандык ырааттарды талдоонун жаңы ыкмаларын компьютердик илимде, криптографияда жана эсептөө процесстерин оптималдаштырууну талап кылган башка тармактарда колдонуу мүмкүнчүлүгүндө болгон

Негизги сөздөр: табигый сандар; так сандын потенциалы; математикалык индукция; ырааттуулукту талдоо; ачык маселе