UDC 004.946
DOI: https://doi.org/10.52754/16948645_2024_1(4)_49

AXIOMATIZATION OF MOTION IN VIRTUAL TOPOLOGICAL SPACES

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Abstract. *The goal of this paper is to survey problems of axiomatization in general, to survey of axiomatization of motion in various spaces and to present a system of axioms to present controlled motion of stretched objects with bounded velocity. Survey of peculiarities of axiomatization in mathematics based on works by A.A. Borubaev and G.M. Kenenbaeva, axiomatization of kinematical spaces is presented in the paper. A new notion of generalized kinematical space is defined: given a family of connected sets of a kinematical space (passes) and a family of connected (isomorphic) sets (things); "passes" contain continuous sequences of "objects".*

Keywords: *axiomatization, topological space, kinematical space, virtual space, velocity, motion.*

ЭЛЕСТЕТИЛГЕН ТОПОЛОГИЯЛЫК МЕЙКИНДИКТЕРДЕ КЫЙМЫЛДООНУ АКСИОМАЛАШТЫРУУ

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Аннотация. *Макаланын максаты – жалпысынан аксиомалаштыруу маселелерин карап чыгуу, ар түрдүү мейкиндиктерде кыймылдын аксиомалаштыруусун карап чыгуу, мейкиндикте чектелген ылдамдыктагы объекттердин башкарылуучу кыймылы үчүн аксиомалардын системасын көрсөтүү. Макалада А.А. Борубаев менен Г.М. Кененбаеванын эмгектеринин негизинде математикадагы аксиомалаштыруунун өзгөчөлүктөрү жана кинематикалык мейкиндиктерди аксиомалаштыруу боюнча жалпы маалымат көрсөтүлгөн. Жалпыланган кинематикалык мейкиндиктин жаңы түшүнүгү аныкталды: кинематикалык мейкиндиктеги көптүктөрдүн топтому (өтмөктөр) жана көптүктөрдүн (объекттердин) топтомдору (изоморфтук) берилет; "Өтмөктөр" "объекттердин" үзгүлтүксүз ырааттуулугун камтыйт.*

Ачкыч сөздөр: *аксиомалаштыруу, топологиялык мейкиндик, кинематикалык мейкиндик, элестетилген мейкиндик, ылдамдык, кыймылдоо.*

АКСИОМАТИЗАЦИЯ ДВИЖЕНИЯ В ВИРТУАЛЬНОМ ТОПОЛОГИЧЕСКОМ ПРОСТРАНСТВЕ

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Аннотация. *Цель статьи – обзор проблем аксиоматизации в целом, обзор аксиоматизации движения в различных пространствах, представление системы аксиом для управляемого движения объектов в пространстве с ограниченной скоростью. В статье представлены обзор особенностей* *аксиоматизации в математике на основе работ А.А. Борубаева и Г.М. Кененбаевой и аксиоматизация кинематических пространств. Определено новое понятие обобщенного кинематического пространства: даны семейство множеств в кинематическом пространстве (проходов) и семейство (изоморфных) множеств (объектов); «проходы» содержат непрерывные последовательности «объектов».*

Ключевые слова: *аксиоматизация, топологическое пространство, кинематическое пространство, виртуальное пространство, скорость, движение.*

1. Introduction In [1] virtual reality was defined as a computer presentation of various spaces known in mathematics in ways close to ways of presenting real (3D-Euclidean) space. This initiated new capacities for investigation.

Nevertheless, in various publications only real space is considered. For instance, [2]: *the virtual reality technology, which has provided a powerful tool for people to experience the virtual world.*

The goal of this work is to survey problems of axiomatization in general, to survey of axiomatization of motion in various spaces and to present a system of axioms to present controlled motion of stretched objects with bounded velocity.

The second section presents a survey of axiomatization. A.A.Borubaev [3] considered ideas and axiomatization of topology and uniform topology. On this base, in the series of works [4-6] a general survey of mathematics was proposed: firstly, some ideas appeared, effects and phenomena had been discovered; further, systems of axioms were developed.

The third section contains a survey of axiomatization of controlled motion of points in a topological space.

The fourth section presents controlled motion of stretched objects with bounded velocity. Topological structures on sets are built by introducing families of subsets meeting some properties. To generalize the notion of a kinematical space we propose to use a family of subsets having "length" (we will call them "passes") and a family of subsets (we will call them "things") which are to be moved along "passes".

2. Survey of axiomatization in mathematics. We cite [3]: *Axiomatization of the notion of continuity had led to the notion of a topological space. There were two ways of axiomatization of the notion of uniform continuity: 1) through the proximity relation of two sets A and B (distance(A,B) is zero in a metric space) as development of P.S. Alexandroff's and K. Kuratowski's viewpoint on a topological space; 2) through axiomatization of properties of the system of ε-neighborhoods in a metric space as development of Hausdorff's viewpoint. The first way had led to the construction of proximity spaces (V.A. Efremovich), the analysis of proximity spaces was held by Ju.M. Smirnov, the second way had led to the construction of uniform spaces (A. Weyl).*

The first systematic exposition of the theory of uniform spaces in terms of entourages was given in Bourbaki's book. Another, but equivalent to the previous concept of a uniform space and defined in terms of a family of coverings was introduced and studied by Tukey. Later, a broad and important study of uniform spaces in the terms of coverings was carried by Yu.M. Smirnov. Isbell's book, in which the theory of uniform spaces got an important development, was also written in terms of the coverings. One can see that the uniform spaces can also be described in terms of pseudometrics …; in terms of metrics over semifields …; in terms of equivalent nets … and small sets … and others.

On the base of this, in the series of works [4-6] a general survey of mathematics was done: firstly, some ideas appeared, effects and phenomena had been discovered; further, systems of axioms were developed. As different systems of axioms codify the same idea, they are equivalent (and proof of their equvalence.

3. Survey of axiomatization of motion of points. The first idea was Gauss's notion of shortest ways along any smooth surfaces (geodesic lines).

As a codifying the ancient idea of controlled motion with bounded velocity, the notion of kinematical space was introduced [1].

Definition 3.1. *A computer program is said to be a **presentation** of a computer kinematical space if:*

P1) there is an (infinite) metrical space X of points and a set X_1 of display-presentable points being sufficiently dense in X;

P2) the user can pass from any point x_1 in X_1 to any other point x_2 by a sequence of adjacent points in X_1 by their will;

P3) the minimal time to reach x_2 from x_1 is (approximately) equal of the minimal time to reach x_2 from x_1.

*The space X is said to be a **kinematic space**; the space X_1 is said to be a **computer kinematic space**; this minimal time is said to be the **kinematical distance** ρ_X between x_1 and x_2; a sequence of adjacent points is said to be a **route**. Passing to a limit as X_1 tends to X we obtain the following.*

*There is a set K of **routes**; each route M, in turn, consists of the positive real number T_M (**time** of route) and the function $m_M : [0, T_M] \to X$ (**trajectory** of route);*

(K1) For $x_1 \neq x_2 \in X$ there exists such $M \in K$ that $m_M(0) = x_1$ and $m_M(T_M) = x_2$, and the set of values of such T_M is bounded with a positive number below;

(K2) If $M= \{T_M , m_M(t)\} \in K$ then the pair $\{T_M , m_M (T_M - t)\}$ is also a route of K (the reverse motion with same speed is possible); (cf. P3).

(K3) If $M= \{T_M, m_M(t)\} \in K$ and $T^* \in (0, T_M)$ then the pair: T^* and function $m^*(t)=m_M(t)$ $(0 \le t \le T^*)$ is also a route of K (one can stop at any desired moment);

(K4) concatenation of routes for three distinct points x_1, x_2, x_3.

*If there exists a kinematic consistent with the given metric then the metric space is said to be **kinematizable**.*

A similar definition also based on the notion of path was proposed in [7].

Denote the set of connected subsets of *R* as *In*. A *path* is a continuous map $\gamma : In \to X$ (a topological space).

The following definition is composed of some definitions in [7] reduced to a "a priori" bounded, path-connected space *X*.

Definition 3.2 (briefly)**.** *A length structure in X consists of a class A of admissible paths together with a function (length) $L: A \to R_+$.*

(A1) The class A is closed under restrictions: if $\gamma \in A$, $\gamma : [a, b] \to X$ and $[u, v] \subset [a, b]$ then the restriction $\gamma|_{[u, v]} \in A$ and the function L is continuous with respect to u,v;

(A2) The class A is closed under concatenations of paths and the function L is additive correspondingly. If a path $\gamma : [a, b] \to X$ is such that its restrictions γ_1, γ_2 to [a, c] and [c, b] belong to A, then so is γ.

(A3) The class A is closed under linear reparameterizations and the function L is invariant correspondingly: for a path $\gamma \in A$, $\gamma : [a, b] \to X$ and a homeomorphism $\varphi : [c, d] \to [a, b]$ of the form $\varphi(t) = \alpha t + \beta$, the composition $\gamma(\varphi(t))$ is also a path.

(A4) [similar to (Kl)].

The metric in X is defined as

$\rho_L(z_0, z_1) := inf\{L(\gamma) \mid \gamma : [a, b] \to X; \gamma \in A; \gamma(a) = z_0; \gamma(b) = z_1\}$.

4. Axiomatization of motion of points with bounded velocity. We [8] proposed controlled motion of stretched sets in topological spaces with bounded velocity based on motion of points as [1].

We propose more general definition.

Consider the following task. Let there be a "thing" and "obstacles". It is necessary to move the thing to another place. Is it possible? If "yes" then in what minimal time it can be done?

The following definition improves one in [9].

Definition 4.1. Let there be a family P of connected subsets of the kinematical space X (**passes**); each pass has the positive **length (time)** and a family Q of connected (isomorphic) subsets of the set X (**things**). [i.e. a thing moves along a pass].

(G1) For each $x \in p \in P$ there exists such $q \in Q$ that $x \in q$ [a thing can be in each place of a pass].

(G2) For each $x_1 \neq x_2 \in X$ there exists such pass $p \in P$ that $x_1, x_2 \in P$ and the set of lengths of such p is bounded with a positive number below; this infimum is said to be the **generalized kinematical distance** ρ_X between points x_1 and x_2.

(G3) For each $q_1 \neq q_2 \in Q$ there exists such pass $p \in P$ that $q_1, q_2 \in P$ and they are continuously connected by elements of Q; the set of lengths of such p is bounded with a positive number below; this infimum is said to be the **generalized kinematical distance** ρ_X between things p_1 and p_2.

(G4) If $x_1, x_2 \in p_1$ and $x_2, x_3 \in p_2$ then there exists such pass $p_3 \in P$ that $x_1, x_2, x_3 \in p_3$ and $\text{length}(p_3) \leq \text{length}(p_1) + \text{length}(p_2)$.

The space X is said to be a **generalized kinematic space**.

If $Q=X$ then Definition 4.1 generalizes Definition 3.1.

5. Conclusion. We hope that the new definitions in this paper would provide effective computer presentations for motion of things in virtual and real spaces.

References

1. Борубаев А.А., Панков П.С. Компьютерные представления кинематических топологических пространств. – Бишкек: КГНУ, 1999.
2. Liu X., Zhang J., Hou G., Wang Z. Virtual Reality and Its Application in Military // Conference Series Earth and Environmental Science, 2018, Vol. 170, issue 3. – P. 32-41.
3. Борубаев А.А. Равномерная топология. - Бишкек: Илим, 2013.
4. Кененбаева Г.М. Обзор эффектов и явлений в различных разделах математики // Вестник Жалал-Абадского государственного университета, 2016, № 1(32). – С. 46-51.
5. Pankov P., Kenenbaeva G. Effect of "numerosity" and other effects in mathematics // Abstracts of the Third BULLETIN OF OSH STATE UNIVERSITY "Actual problems of the theory of control, topology and operator equations" / Ed. by Academician A. Borubaev. – Bishkek: Kyrgyz Mathematical Society, 2017. – P. 87.
6. Pankov P. S., Kenenbaeva G. M. Hypothesis on effect of "numerosity" and other effects in mathematics // Наука, новые технологии и инновации Кыргызстана. – Бишкек, 2017, № 5. – С. 60-62.
7. Burago D., Burago Yu., Ivanov S. A Course in Metric Geometry // Graduate Studies in Mathematics, Volume 33, American Mathematical Society. Providence, Rhode Island, 2001.
8. Zhoraev A.H. Motion of sets and orientation dimension of kinematical spaces // Abstracts of the VI Congress of the Turkic World Mathematical Society. – Astana: L.N. Gumilyov Eurasian National University, 2017. – P. 124.
9. Zhoraev A.H. Axiomatization of kinematical spaces // Herald of Institute of Mathematics of NAS of KR, 2021, No. 1. – P. 16-21.