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DIFFERENTIAL *l*-CATCH AND *l*-ESCAPE GAMES IN THE CASE OF NON-STATIONARY GEOMETRIC CONSTRAINTS ON CONTROLS

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Abstract: *This paper is devoted to the l-catch and l-escape differential games with two players, called pursuer and evader, whose controls adhere to non-stationary geometric constraints of various types. Such problems are quite relevant for the processes where the rates of control parameters fluctuate consistently during the time. First, the pursuit problem is discussed and a pursuer strategy guaranteeing the l-catch is defined using the method of Chikrii's resolving functions. Then, the evasion problem is dealt with by means of a specific control function of evader.*

Keywords: *Differential game, l-catch, evasion, pursuer, evader, geometric constraint, strategy, guaranteed time of l-catch.*

ДИФФЕРЕНЦИАЛЬНИХ ИГР *l*-ПОИМКИ И *l*-УБЕГАНИЯ В СЛУЧАЕ НЕСТАЦИОНАРНЫХ ГЕОМЕТРИЧЕСКИХ ОГРАНИЧЕНИЯХ НА УПРАВЛЕНИЯМИ

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Аннотация: *В этой статье рассмотрена проблемы чем l-поимка и l-убегания для дифференциальных игр с двумя игроками, называется преследователь и убегающий, управление которых придерживается нестационарного геометрические связи различных типов. Такие проблемы весьма актуальны для процессы, в которых скорости управляющих параметров постоянно колеблются во времени. Мы построили стратегию сходимости на основе метода разрешающих функции А.А.Чикрия для преследователя и представили новые достаточные условия l-поимки. Здесь, под l-захватом мы понимаем момент, когда преследователь приближаться к убегающему на расстояние l>0. В задаче об уклонении мы определили стратегию, гарантирующую уклонение убегающего от преследователя на расстояние большее, чем l>0. Кроме того, показаны новые достаточные условия уклонения.*

Ключевые слова: *дифференциальная игра, l-поимка, убегания, преследователь, убегающий, геометрическое ограничение, стратегия, гарантированное время l-поимка.*

1. Introduction. Differential Theory of Differential Games looks into conflict problems in systems which are expressed by differential equations. As a result of the growth of Pontryagin's maximum principle, it became apparent that there was a link between optimal control theory and differential games. Actually, problems of differential game describe a generalization of optimal control problems in cases where there are more than one player.

The study of differential games was initiated by American mathematician R. Isaacs. His research was published in the form of a monograph [1, p. 340] in 1965, in which a great number of examples were considered, and theoretical que stions were only affected. Differential games have been one of the basic research fields since the 1960th and their fundamental results were gained by L.S. Pontryagin [2, p. 551], N.N. Krasovsky [3, p. 517].

N.N .Krasovsky and his followers estimated the quality of pursuit by the time span from the initial instant of the process up to the ℓ-capture instant ($\ell > 0$). This method is based on the extreme sighting rule which gives in a number of cases of the equilibrium point. The method was conclusively formulated in the monograph by N.N. Krasovsky.

The problem for the case of ℓ-approach [5, p. 272] was first studied by Indian mathematician Ramchundra. Analogous effects in the case of geometrical constraint were considered in the works of Pshenichnyi [6, p. 484-485], Petrosyan [4, p. 31-38], Satimov [7, p. 203], Azamov [8, p. 38-43], Samatov [9, 10] and others studied that problem, and interesting results were obtained by them. In the work of Petrosyan and Dutkevich [4, p. 31-38], the l-capture problem was investigated for the players moving at the limited velocities by coordinates on the plane and also, a lifeline game was solved by geometrical method. Later on, by virtue of Chikrii's method of resolving functions, B.T.Samatov [9, p. 907-921; 10, p. 94-107] solved the problem of group pursuit for the case of l-capture in a simple motion of the players under integral constraints on controls. In [11, p. 574-579], Khaidarov considered the problem of positional l-capture of one evader by a group of pursuers provided that each of the players has a simple movement.

In the paper, we have considered the l-catch and l-escape problems in a differential game with one evader and one pursuer, whose controls are subject to non-stationary geometrical constraints. In the l-catch problem, an approach strategy is constructed for a pursuer and sufficient conditions of l-catch are obtained. In the l-escape problem, a specific strategy is suggested for an evader and sufficient conditions of evasion are found. Furthermore, it is shown how the distance between the players changes during the evasion game.

2. Statement of problems. We will consider the differential game which includes two players *P* (Pursuer) and *E* (Evader) whose state vectors are x and y, and whose velocity vectors are u and v, respectively in the space $\mathbb{R}^n$. Let, in this consideration, the motion dynamics of *P* and *E* be described by the differential equations

$$P: \qquad \dot{x} = u, \qquad x(0) = x_0, \tag{1}$$

$$E: \qquad \dot{y} = u, \qquad y(0) = y_0 \tag{2}$$

correspondingly, where $x, y, u, v \in \mathbb{R}^n$, $n \geq 2$; x_0, y_0 are the initial states of the players for which it is presumed that $|x_0 - y_0| > l$, $l > 0$; the velocity vectors u and v act as control parameters of the players respectively, and they depend on time $t \geq 0$.

The controls u and v are regarded as measurable functions $u(\cdot):[0,+\infty) \to \mathbb{R}^n$ and $v(\cdot):[0,+\infty) \to \mathbb{R}^n$ accordingly, and they are subject to the constraints

$$|u(t)| \leq \rho a^{-kt} + a^{kt} \text{ for almost every } t \geq 0, \tag{3}$$

$$|v(t)| \leq \sigma a^{-kt} + a^{kt} \text{ for almost every } t \geq 0, \tag{4}$$

where ρ, σ, k are the given positive parametric numbers. Let $U_\rho^{a,k}$ stand for the family of all measurable functions corresponding to (3). Similarly, let the family of all measurable functions satisfying (4) be represented by $V_\sigma^{a,k}$.

Definition 1. The measurable functions $u(\cdot)=\left(u_1(\cdot),...,u_n(\cdot)\right)\in U_\rho^{a,k}$ $\left(v(\cdot)=\left(v(\cdot)=v_1(\cdot),...,v_n(\cdot)\right)\in V_\sigma^{a,k}\right)$ is called an admissible control of the player *P* (of the player *E*).

If $u(\cdot)\in U_\rho^{a,k}$ and $v(\cdot)\in V_\sigma^{a,k}$, then the solutions to Cauchy's problems (1) and (2) are

$$x(t)=x_0+\int_0^t u(s)ds, \qquad y(t)=y_0+\int_0^t v(s)ds$$

and the pairs $\left(x_0,u(\cdot)\right)$ and generate the motion trajectories ***of the player*** **P** ***and*** **E** ***appropriately.***

The main target of the player P is to gain ground the player E at the distance $l>0$ (l-catch problem), i.e., to achieve the relation

$$\left|x(\theta)-y(\theta)\right|\le l \tag{5}$$

at some finite time $\theta>0$. Whereas the objective of the player E is to avoid the occurrence of (5) (l-escape problem, i.e., to keep the inequality

$$\left|x(t)-y(t)\right|>l \tag{6}$$

for all $t\ge 0$ or, if it is impossible, to put off the time of the occurred of (5).

There is no doubt that control functions depending only on the time-parameter $t, t\ge 0$ are not sufficient to solve the l-catch problem, and hence the acceptable types of controls should be strategies for the player P.

We will introduce the following denotations for the sake of convenience:

$$z(t)=x-y, \quad z_0=x_0-y_0.$$

Then equation (1) and (2) come to the unique Cauchy problem in the form

$$\dot z=u-v, \quad z(0)=z_0. \tag{7}$$

3. The main results.

Definition 2. For $\rho\ge\sigma$, we call the function

$$\mathrm{u}\left(z_0,t,v\right)=v+\lambda\left(z_0,t,v\right)\left(m\left(z_0,t,v\right)-z_0\right)$$

(8)

the *l-approach strategy* or Π_l *-strategy* for *P* in the differential game (1)-(4), where $\lambda(v,z_0)=\frac{1}{\left|z_0\right|^2-l^2}\left[\langle v,z_0\rangle+\varphi(t)l+\sqrt{\left(\langle v,z_0\rangle+\varphi(t)l\right)^2+\left(\left|z_0\right|^2-l^2\right)\left(\varphi^2(t)-\left|v\right|^2\right)}\right]$,

$\varphi(t)=\rho a^{-kt}+a^{kt}$, $m\left(z_0,t,v\right)=-\frac{v-\lambda\left(z_0,t,v\right)z_0}{\left|v-\lambda\left(z_0,t,v\right)z_0\right|}l$.

Here $\langle v,z_0\rangle$ is scalar product of the vectors v and z_0 in $\mathbb{R}^n$. Moreover, the function $\lambda(z_0,t,v)$ is usually called the *resolving function*. Below we will provide some important properties for the strategy (8) and the resolving function $\lambda(z_0,t,v)$.

Proposition 1. If $\rho \geq \sigma$ holds, then the strategy (8) is defined and continuous for any v, $|v| \leq \beta$, and the equality $|\mathrm{u}(z_0, t, v)| = \varphi(t)$ holds during the *l*-catch game.

Proposition 2. If $\alpha \geq \beta$ is valid, then the function $\lambda(v, z_0)$ is defined, non- negative and continuous for any v, $|v| \leq \beta$, and it is bounded as

$$\left(\varphi(t)-|v(t)|\right)/\left(|z_0|-l\right) \leq \lambda(z_0,t,v) \leq \left(\varphi(t)+|v(t)|\right)/\left(|z_0|-l\right). \tag{9}$$

Definition 3. It is said that the Π_l -strategy (8) *guarantees to occur l*-catch on time interval $\left[0, T\left(z_0, v(\cdot)\right)\right]$ if, for any $v(\cdot) \in V_\sigma^{a,k}$:

a) their exists an instant $t_* \in \left[0, T\left(z_0, v(\cdot)\right)\right]$ at which $|z(t_*)| \leq l$ is satisfied;

b) an inclusion $\mathrm{u}(v, z_0) \in U_\rho^{a,k}$ is fullfilled on the interval $[0, t_*]$, where we say the number $T\left(z_0, v(\cdot)\right)$ a *guaranteed time of l*-catch**.**

Theorem 1. If one of the following conditions holds in differential game (1) – (4), that is, 1. $0 < a < 1,\ \ \rho > \sigma$ or 2. $a > 1,\ \ \rho > \sigma + k(|z_0| - l)\ln a$, then Π_l -strategy (8) guarantees to occur l-catch on the time $T\left(z_0, v(\cdot)\right) \leq T_l$ in the *l*-catch problem (1)-(4), where

$$T_l = \frac{1}{k}\log_a \frac{\rho-\sigma}{\rho-\sigma-\left(|z_0|-l\right)k\ln a}.$$

Definition 4. We call the control function

$$v_*(t) = -\left(\sigma a^{-kt} + a^{kt}\right)\frac{z_0}{|z_0|} \tag{10}$$

a strategy of the player *E* in the game (1)-(4).

Definition 5. We say that the strategy $v_*(t)$ is *winning* if, for any control $u(\cdot) \in U_\rho^{a,k}$**,** *the solution* $z(t)$ *of*

$$\dot{z} = u(t) - v_*(t),\ z(0) = z_0 \tag{11}$$

fulfills the inequality (6) for all t, $t \geq 0$.

Theorem 2. If one of the following conditions holds:

1. $0 < a < 1,\ \ \rho \leq \sigma$ *; or 2.* $a > 1,\ \ \rho \leq \sigma + k(|z_0| - l)\ln a$,

then in the differential game (1) – (4), the *l*-escape problem is solved by the strategy of the player *E* (10) and a change function between the players will be in the following form:

$$E(t) = |z_0| - l + \frac{\sigma-\rho}{k\ln a}\left(1 - a^{-kt}\right).$$

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