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## МАТЕМАТИКА

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# NONLINEAR TWO-POINT BOUNDARY VALUE PROBLEM FOR A SECOND ORDER IMPULSIVE SYSTEM OF INTEGRO-DIFFERENTIAL EQUATIONS WITH MIXED MAXIMA

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***Abstract:*** *A two-point nonlinear boundary value problem for a second order system of ordinary integro-differential equations with impulsive effects and mixed maxima is investigated. By applying some transformations is obtained a system of nonlinear functional integral equations. The existence and uniqueness of the solution of the nonperiodic two-point boundary value problem are reduced to the one valued solvability of the system of nonlinear functional integral equations in Banach space* $PC\left([0,T],\mathbb{R}^n\right)$. *The method of successive approximations in combination with the method of compressing mapping is used in the proof of one-valued solvability of nonlinear functional integral equations.*

***Keywords:*** *Second order system, impulsive integro-differential equations, two-point nonlinear boundary value conditions, mixed maxima, successive approximations, existence and uniqueness of solution.*

# НЕЛИНЕЙНАЯ ДВУХТОЧЕЧНАЯ КРАЕВАЯ ЗАДАЧА ДЛЯ ИМПУЛЬСНОЙ СИСТЕМЫ ИНТЕГРОДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ВТОРОГО ПОРЯДКА СО СМЕШАННЫМИ МАКСИМУМАМИ

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***Аннотация:*** *Исследуется двухточечная нелинейная краевая задача для системы обыкновенных интегро-дифференциальных уравнений второго порядка с импульсными эффектами и смешанными максимумами. Путем применения некоторых преобразований получается система нелинейных функциональных интегральных уравнений. Существование и единственность решения непериодической двухточечной краевой задачи сводятся к однозначной разрешимости системы нелинейных функциональных интегральных уравнений в Банаховом пространстве* $PC\left([0,T],\mathbb{R}^n\right)$. *Метод последовательных приближений в сочетании с методом сжимающих отображений используется при доказательстве однозначной разрешимости нелинейных функциональных интегральных уравнений.*

***Ключевые слова:*** *система второго порядка, импульсные интегро-дифференциальные уравнения, двухточечные нелинейные краевые условия, смешанные максимумы, последовательные приближения, существование и единственность решения.*

**1. Introduction.**

Mathematical model of many problems of modern sciences, technology and economics are described by differential and integro-differential equations, the solutions of which are functions with first kind discontinuities at fixed or non-fixed times. Such differential and integro-differential equations are called equations with impulsive effects. A lot of publications of studying on differential and integro differential equations with impulsive effects, describing many natural and technical processes, are appearing (see, for examples, [1–20]. Two-point and multi-point boundary value problems for the differential and integro-differential equations are studied by many authors (see, for example [21–24]. Second-order differential equations with nonlocal boundary value conditions and impulsive effects are almost not studied. The fact is that the reduction of such a problem to an equivalent functional integral equation has difficulties. In this paper, we investigate a two-point nonlinear boundary value problem for a system of second order integro-differential equations with impulsive effects, nonlinear kernel depending on construction of mixed maxima. The questions of existence and uniqueness of the solution to the nonlinear two-point boundary value problem are investigated. We note that the differential and integro-differential equations with mixed maxima have singularity in studying of the questions of solvability. Moreover, the jumpiness of solutions is a natural thing for differential equations with mixed maxima [25].

On the interval $[0,T]$ for $t \neq t_i,\ i=1,2,...,p$ we consider the questions of existence and constructive methods of calculating of the unique solutions of the second order system of nonlinear ordinary integro-differential equations with impulsive effects and maxima

$$x''(t) = f\left( t, x(t), \int_0^T \Theta\left(t, s, \max\left\{ x(\tau) \middle| \tau \in \left[\lambda_1(s) :|: \lambda_2(s)\right]\right\}\right) ds \right), \tag{1}$$

where $t \neq t_i,\ i=1,2,...,p,\ \ 0=t_0<t_1<...<t_p<t_{p+1}=T,\ \ x \in X$ , $X$ is the closed bounded domain in the space $\mathbb{R}^n$ , $f(t,x,y) \in \mathbb{R}^n$, $0<\lambda_i(t)<T,\ i=1,2,$

$$\left[\lambda_1(t) :|: \lambda_2(t)\right] = \left[\min\left\{\lambda_1(t), \lambda_2(t)\right\}; \max\left\{\lambda_1(t), \lambda_2(t)\right\}\right],\ \max_{0\le t\le T} \int_0^T \left| \Theta(t,s,x) \right| ds < \infty.$$

The equation (1) we study with nonlinear conditions

$$A_1(t)x(0^+) + B_1(t)x(T^-) = C_1(t, x(t)), \tag{2}$$

$$A_2(t)x'(0^+) + B_2(t)x'(T^-) = C_2(t, x(t)) \tag{3}$$

and nonlinear impulsive effect

$$x\left(t_i^+\right) - x\left(t_i^-\right) = F_i\left(x\left(t_i\right)\right), \qquad i=1,2,...,p, \tag{4}$$

$$x'\left(t_i^+\right) - x'\left(t_i^-\right) = G_i\left(x\left(t_i\right)\right), \qquad i=1,2,...,p, \tag{5}$$

where $A_i(t), B_i(t)$ are $n \times n$-dimensional matrix-functions, $C_i(t, x(t)) \in \mathbb{R}^n$ is nonlinear vector-function, $i=1,2$, $F_i, G_i \in \mathbb{R}^n$, $0<\lambda_i(t)<T,\ i=1,2,$ $x\left(t_i^+\right) = \lim_{\nu \to 0^+} x\left(t_i+\nu\right)$, $x\left(t_i^-\right) = \lim_{\nu \to 0^-} x\left(t_i-\nu\right)$ are right-hand side and left-hand side limits of function $x(t)$ at the point $t=t_i$, respectively.

$C\left([0,T], \mathbb{R}^n\right)$ is the notation of the Banach space, which consists continuous vector function

$x(t)$, defined on the segment $[0,T]$, with the norm

$$\| x \| = \sqrt{\sum_{j=1}^{n} \max_{0 \le t \le T} \left| x_j(t) \right|}.$$

$PC\left([0,T],\mathbb{R}^n\right)$ is the notation of the following linear vector space

$$PC\left([0,T],\mathbb{R}^n\right) = \left\{ x:[0,T] \to \mathbb{R}^n;\ x(t) \in C\left(\left(t_i, t_{i+1}\right], \mathbb{R}^n\right),\ i = 1,\dots,p \right\},$$

where $x\left(t_i^+\right)$ and $x\left(t_i^-\right)$ $(i = 0,1,\dots,p)$ exist and are bounded; $x\left(t_i^-\right) = x\left(t_i\right)$. Note, that the linear vector space $PC\left([0,T],\mathbb{R}^n\right)$ is Banach space with the following norm

$$\| x \|_{PC} = \max\left\{ \| x \|_{C((t_i, t_{i+1}])},\ i = 1,2,\dots,p \right\}.$$

**2. Formulation of problem.**

To find the function $x(t) \in PC\left([0,T],R^n\right)$, which for all $t \in [0,T],\ t \ne t_i,\ i = 1,2,\dots,p$ satisfies the second-order integro-differential equation (1), nonlinear two point conditions (2), (3) and for $t = t_i\ i = 1,2,\dots,p,\ 0 < t_1 < t_2 < \dots < t_p < T$ satisfies the nonlinear limit conditions (3), (4).

**3. Reduction to nonlinear system of functional integral equations.**

Let the function $x(t) \in PC\left([0,T],\mathbb{R}^n\right)$ is a solution of the second order boundary value problem (1)-(5). Then, integrating the integro-differential equation (1) one time on the intervals: $\left(0,t_1\right], \left(t_1,t_2\right], \dots, \left(t_p, t_{p+1}\right]$, we obtain:

$$\int_0^{t_1} f(x)\,ds = \int_0^{t_1} x''(s)\,ds = x'(t_1^-) - x'(0^+),\ t \in \left(0,t_1\right],$$

$$\int_{t_1}^{t_2} f(s)\,ds = \int_{t_1}^{t_2} x''(s)\,ds = x'\left(t_2^-\right) - x'\left(t_1^+\right),\ t \in \left(t_1,t_2\right],$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\int_{t_p}^{t_{p+1}} f(s)\,ds = \int_{t_p}^{t_{p+1}} x''(s)\,ds = x'\left(t_{p+1}^-\right) - x'\left(t_p^+\right),\ t \in \left(t_p, t_{p+1}\right],$$

where for convenience, we put

$$f(t) = f\left( t, x(t), \int_0^T \Theta\left(t, s, \max\left\{ x(\tau) \middle| \tau \in [\lambda_1(s), \lambda_2(s)] \right\}\right) ds \right).$$

Hence, taking $x'(0^+) = x'(0),\ x'(t_{k+1}^-) = x'(t)$ into account, on the interval $(0,T]$ we have

$$\int_0^t f(s)\,ds = \left[ x'\left(t_1\right) - x'\left(0^+\right) \right] + \left[ x'\left(t_2\right) - x'\left(t_1^+\right) \right] + \dots + \left[ x'(t) - x'\left(t_p^+\right) \right] =$$

$$= -x'(0) - \left[ x'\left(t_1^+\right) - x'\left(t_1\right) \right] - \left[ x'\left(t_2^+\right) - x'\left(t_2\right) \right] - \dots - \left[ x'\left(t_p^+\right) - x'\left(t_p\right) \right] + x'(t).$$

Taking into account the condition (5), the last equality we rewrite as

$$x'(t) = x'(0) + \int_0^t f(s)\,ds + \sum_{0<t_i<t} G_i\left(x\left(t_i\right)\right). \tag{6}$$

We subordinate the function $x'(t) \in PC\left([0,T], \mathbb{R}^n\right)$ in presentation (6) to satisfy the nonlinear two-point boundary condition (3):

$$x'(T) = x'(0) + \int_0^T f(s)\,ds + \sum_{0<t_i<T} G_i\left(x\left(t_i\right)\right). \tag{7}$$

Substituting (7) into condition (3), we find $x'(0)$ as follows:

$$x'(0) = Q_2^{-1}(t)\left[C_2(t, x(t)) - B_2(t)\int_0^T f(s)\,ds - B_2(t)\sum_{0<t_i<T} G_i\left(x\left(t_i\right)\right)\right], \tag{8}$$

where $\det Q_2(t) \neq 0$, $Q_2(t) = A_2(t) + B_2(t)$.

Substituting (8) into presentation (6), we obtain:

$$x'(t) = Q_2^{-1}(t)\left[C_2(t, x(t)) - B_2(t)\int_0^T f(s)\,ds - B_2(t)\sum_{0<t_i<T} G_i\left(x\left(t_i\right)\right)\right] +$$

$$+\int_0^t f(s)\,ds + \sum_{0<t_i<t} G_i\left(x\left(t_i\right)\right). \tag{9}$$

Then, integrating integro-differential equation (9) one time on the intervals $\left(0, t_1\right]$, $\left(t_1, t_2\right]$,..., $\left(t_p, t_{p+1}\right]$ and taking $x'(0^+) = x'(0)$, $x'(t_{k+1}^-) = x'(t)$ into account, on the interval $(0, T]$ we have

$$\int_0^t Q_2^{-1}(s)\left[C_2(s, x(s)) - B_2(s)\int_0^T f(\theta)\,d\theta - B_2(s)\sum_{0<t_i<T} G_i\left(x\left(t_i\right)\right)\right]ds +$$

$$+\int_0^t\left[\int_0^s f(\theta)\,d\theta + \sum_{0<t_i<s} G_i\left(x\left(t_i\right)\right)\right]ds =$$

$$= \left[x\left(t_1\right) - x\left(0^+\right)\right] + \left[x\left(t_2\right) - x\left(t_1^+\right)\right] + \ldots + \left[x(t) - x\left(t_p^+\right)\right] =$$

$$= -x(0) - \left[x\left(t_1^+\right) - x\left(t_1\right)\right] - \left[x\left(t_2^+\right) - x\left(t_2\right)\right] - \ldots - \left[x\left(t_p^+\right) - x\left(t_p\right)\right] + x(t). \tag{10}$$

Taking into account the nonlinear impulsive condition (4), from the last equality (10) we derive

$$x(t) = x(0) + \int_0^t Q_2^{-1}(s)\left[C_2(s, x(s)) - B_2(s)\int_0^T f(\theta)\,d\theta - B_2(s)\sum_{0<t_i<T} G_i\left(x\left(t_i\right)\right)\right]ds +$$

$$+\int_0^t\left[\int_0^s f(\theta)\,d\theta + \sum_{0<t_i<s} G_i\left(x\left(t_i\right)\right)\right]ds + \sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right). \tag{11}$$

Applying the two-point nonlinear condition (2) to the equation (11), we find the value of $x(0)$ as follows:

$$x(0)=Q_1^{-1}(t)C_1(t,x(t))-\int_0^T Q_1^{-1}(t)B_1(t)Q_2^{-1}(s)C_2(s,x(s))ds+$$

$$+\int_0^T Q_1^{-1}(t)B_1(t)Q_2^{-1}(s)B_2(s)\int_0^T f(\theta)\,d\theta ds+$$

$$+\int_0^T Q_1^{-1}(t)B_1(t)Q_2^{-1}(s)B_2(s)\sum_{0<t_i<t} G_i\left(x\left(t_i\right)\right)ds-Q_1^{-1}(t)B_1(t)\int_0^T\int_0^s f(\theta)\,d\theta\,ds-$$

$$-Q_1^{-1}(t)B_1(t)\int_0^T \sum_{0<t_i<t} G_i\left(x\left(t_i\right)\right)ds-Q_1^{-1}(t)B_1(t)\sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right). \quad (12)$$

In getting (12), we used well known formulas, which connected by the name of Dirichlet:

$$\int_0^T g(t,s)\int_0^s f(\theta)\,d\theta ds=\int_0^T f(s)\int_s^T g(t,\theta)\,d\theta\,ds,$$

$$\int_0^T g(t,s)\sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right)ds=\sum_{0<t_i<T}\int_{t_i}^T g(t,s)\,ds\,I_i\left(x\left(t_i\right)\right).$$

Then, we rewrite (12) as follows

$$x(0)=Q_1^{-1}(t)C_1(t,x(t))-\int_0^T V_0(t,s)C_2(s,x(s))ds+$$

$$+\int_0^T V_1(t,s)f(s)\,ds+\sum_{0<t_i<T} V_1(t,t_i)G_i\left(x\left(t_i\right)\right)-Q_1^{-1}(t)B_1(t)\sum_{0<t_i<T} F_i\left(x\left(t_i\right)\right), \quad (13)$$

where $V_0(t,s)=Q_1^{-1}(t)B_1(t)Q_2^{-1}(s),\quad \det Q_1(t)\neq 0,\quad Q_1(t)=A_1(t)+B_1(t),$

$$V_1(t,s)=Q_1^{-1}(t)B_1(t)\left[\int_s^T Q_2^{-1}(\theta)\left[A_2(\theta)+2B_2(\theta)\right]d\theta\right].$$

Substituting (13) into presentation (11), we obtain final view of nonlinear system of functional integral equations:

$$x(t)=J(t;x)\equiv Q_1^{-1}(t)C_1(t,x(t))+\int_0^T W_0(t,s)C_2(s,x(s))ds+$$

$$+\int_0^T W_1(t,s)f\left(s,x(s),\int_0^T \Theta\left(s,\theta,\max\left\{x(\tau)\middle|\tau\in[\lambda_1(\theta),\lambda_2(\theta)]\right\}\right)d\theta\right)ds+$$

$$+\sum_{0<t_i<T} W_1(t,t_i)G_i\left(x\left(t_i\right)\right)+\sum_{0<t_i<T} W_2(t_i)F_i\left(x\left(t_i\right)\right), \quad (14)$$

where

$$W_0(t,s)=\begin{cases}-V_0(t,s), & t\le s\le T,\\ -V_0(t,s)+Q_2^{-1}(s), & 0\le s<t,\end{cases}$$

$$W_1(t,s)=\begin{cases} V_1(t,s), \ t\le s\le T, \\ V_1(t,s)-\int_0^t Q_2^{-1}(\theta)B_2(\theta)\,d\theta+\int_s^t Q_2^{-1}(\theta)\left[A_2(\theta)+B_2(\theta)\right]d\theta, \ 0\le s<t, \end{cases}$$

$$W_2(s)=\begin{cases} -Q_1^{-1}(s)B_1(s), \ t\le s\le T, \\ Q_1^{-1}(s)A_1(s), \quad 0\le s<t. \end{cases}$$

**3. One valued solvability.**

**Theorem.** Suppose that the following conditions are fulfilled:

1). $M_f=\max\limits_{0\le t\le T}\left| f\left(t,0,\int_0^T \Theta(t,s,0)\,ds\right)\right|<\infty;\ M_{C_i}=\max\limits_{0\le t\le T}\left|C_j(t,0)\right|<\infty, \ j=1,2;$

2). $m_F=\max\limits_{i\in\{1,2,...,p\}}\left|F_i(0)\right|<\infty, \ m_G=\max\limits_{i\in\{1,2,...,p\}}\left|G_i(0)\right|<\infty;$

3). For all $t\in[0,T]$, $x,y\in R^n$ holds

$$\left|f(t,x_1,y_1)-f(t,x_2,y_2)\right|\le M_1(t)\left|x_1-x_2\right|+M_2(t)\left|y_1-y_2\right|;$$

4). For all $t,s\in[0,T]^2$, $x\in R^n$ holds

$$\left|\Theta(t,s,x_1)-\Theta(t,s,x_2)\right|\le M_3(t,s)\left|x_1-x_2\right|;$$

5). For all $t\in[0,T]$, $x\in R^n$ holds

$$\left|C_j(t,x_1)-C_j(t,x_2)\right|\le M_{4j}(t)\left|x_1-x_2\right|, \ j=1,2;$$

6). For all $x\in R^n$, $i=0,1,...,p$ hold

$$\left|F_i(x_1)-F_i(x_2)\right|\le m_{1i}\left|x_1-x_2\right|, \ \left|G_i(x_1)-G_i(x_2)\right|\le m_{2i}\left|x_1-x_2\right|;$$

7). $\rho=\chi_1+...+\chi_5<1$, where $\chi_1,...,\chi_5$ are defined by the formulas (18)-(20) below.

Then the two-point boundary value problem (1)-(5) has a unique solution $x(t)\in PC\left([0,T],R^n\right)$. This solution can be founded by the following iterative process:

$$\begin{cases} x^k(t)=J(t;x^{k-1}), \ k=1,2,3,... \\ x^0(t)=0, \ t\in(t_i,t_{i+1}), \ i=0,1,2,...,p. \end{cases} \tag{15}$$

**Proof.** We consider the following operator

$$J:PC\left([0,T];R^n\right)\to PC\left([0,T]\times R^n\right),$$

defined by the right-hand side of equation (14). Applying the principle of contracting operators to (14), we show that the operator $J$, defined by equation (14), has a unique fixed point.

Taking first and second conditions of the theorem, for the first difference of the approximations (15) we have the following estimate

$$\left\| x^1(t)-x^0(t)\right\|\le\max_{0\le t\le T}\left|Q_1^{-1}(t)\right|\cdot\left|C_1(t,0)\right|+\max_{0\le t\le T}\int_0^T\left|W_0(t,s)\right|\cdot\left|C_2(t,0)\right|ds+$$

$$+\max_{0\le t\le T}\int_0^T\left|W_1(t,s)\right|\left| f\left(s,0,\int_0^T\Theta(s,\theta,0)\,d\theta\right)\right|ds+$$

$$+\max_{0\le t\le T}\sum_{i=1}^{p}\left|W_1(t,t_i)\right|\cdot\left|G_i(0)\right|+\sum_{i=1}^{p}\left|W_2(t_i)\right|\cdot\left|F_i(0)\right|\le$$

$$\le\left\|Q_1^{-1}(t)\right\|M_{C_1}+\sigma_0 M_{C_2}+\sigma_{11}M_f+\sigma_{12}m_G+\sigma_2 m_F<\infty, \tag{16}$$

where

$$\sigma_0=\max_{0\le t\le T}\int_0^T\left|W_0(t,s)\right|ds,\quad \sigma_{11}=\max_{0\le t\le T}\int_0^T\left|W_1(t,s)\right|ds,$$

$$\sigma_{12}=\max_{0\le t\le T}\sum_{i=1}^{p}\left|W_1(t,t_i)\right|,\quad \sigma_2=\sum_{i=1}^{p}\left|W_2(t_i)\right|.$$

Then, by the third - sixth conditions of the theorem, for difference of arbitrary consecutive approximations and arbitrary $t\in(t_i,t_{i+1}]$ we have

$$\left\|x^{k+1}(t)-x^k(t)\right\|\le\max_{0\le t\le T}\left|Q_1^{-1}(t)\right|M_{41}(t)\left|x^k(t)-x^{k-1}(t)\right|+$$

$$+\max_{0\le t\le T}\int_0^T\left|W_0(t,s)\right|M_{42}(s)\left|x^k(s)-x^{k-1}(s)\right|ds+$$

$$+\max_{0\le t\le T}\int_0^T\left|W_1(t,s)\right|\Big[M_1(s)\left|x^k(s)-x^{k-1}(s)\right|+$$

$$+M_2(s)\int_0^T M_3(s,\theta)\left|x^k(\theta)-x^{k-1}(\theta)\right|d\theta\Big]ds+$$

$$+\max_{0\le t\le T}\sum_{i=1}^{p}\left|W_1(t,t_i)\right|m_{2i}\left|x^k(t_i)-x^{k-1}(t_i)\right|+\sum_{i=1}^{p}\left|W_2(t_i)\right|m_{1i}\left|x^k(t_i)-x^{k-1}(t_i)\right|.$$

Hence, by the introduced norm in the space $PC([0,T],R^n)$ we obtain

$$\left\|x^k(t)-x^{k-1}(t)\right\|_{PC}\le\rho\cdot\left\|x^{k-1}(t)-x^{k-2}(t)\right\|_{PC}, \tag{17}$$

where $\rho=\chi_1+...+\chi_5$,

$$\chi_1=\max_{0\le t\le T}\left|Q_1^{-1}(t)\right|M_{41}(t),\quad \chi_2=\max_{0\le t\le T}\int_0^T\left|W_0(t,s)\right|M_{42}(s)ds, \tag{18}$$

$$\chi_3=\max_{0\le t\le T}\int_0^T\left|W_1(t,s)\right|\left[M_1(s)+M_2(s)\int_0^T M_3(s,\theta)\,d\theta\right]ds, \tag{19}$$

$$\chi_4=\max_{0\le t\le T}\sum_{i=1}^{p}\left|W_1(t,t_i)\right|m_{2i},\quad \chi_5=\sum_{i=1}^{p}\left|W_2(t_i)\right|m_{1i}. \tag{20}$$

According to the last condition of the theorem, we have $\rho<1$. Therefore, from the estimate (17) follows that

$$\left\| x^k(t) - x^{k-1}(t) \right\|_{PC} < \left\| x^{k-1}(t) - x^{k-2}(t) \right\|_{PC}. \tag{21}$$

It implies from (21) that the operator $J$ on the right-hand side of the equation (14) is contracting. According to fixed point principle in the Banach space $PC\left([0,T],R^n\right)$ and taking into account estimates (16), (17), we conclude that the operator $J$ has a unique fixed point. Consequently, the two-point nonlinear boundary value problem (1)-(5) has a unique solution $x(t) \in PC\left([0,T],R^n\right)$.

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