МАТЕМАТИКА

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DIFFERENTIAL L-CAPTURE AND EVASION GAMES WITH INERTIAL PLAYERS UNDER GEOMETRIC CONSTRAINTS ON CONTROLS

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Abstract. *This paper is devoted to solve l-capture and evasion problems for a differential game of two players, a pursuer and an evader, with inertial motions. We impose geometric constraints on controls of the players. Originally, we devise an l-approach strategy, on the basis of Chikrii's method of resolving functions, for a pursuer and we present new sufficient conditions of l-capture. Here as l-capture, we refer the moment when a pursuer approaches an evader at the range l>0. In the evasion problem we define the strategy guaranteeing an evader to diverge from a pursuer at the distance greater than l>0. Besides that, new sufficient conditions of evasion have been shown.*

Keywords: *Differential game, l-capture, evasion, pursuer, evader, geometric constraint, strategy, guaranteed time of l-capture.*

ДИФФЕРЕНЦИАЛЬНИХ ИГР *l*-ПОИМКИ И УБЕГАНИЯ С ИНЭРЦИОННЫМИ ИГРОКАМИ ПРИ ГЕОМЕТРИЧЕСКИХ ОГРАНИЧЕНИЯХ НА УПРАВЛЕНИЯМИ

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Аннотация: *В этой статье рассмотрена проблемы чем l-поимка и убегания для дифференциальных игр с двумя игроками, называется преследователь и убегающий, имеет инерционные движения. Мы наложили геометрические ограничения на управление игроками. Мы построили стратегию сходимости на основе метода разрешающих функции А.А.Чикрия для преследователя и представили новые достаточные условия l-поимки. Здесь, под l-захватом мы понимаем момент, когда преследователь приближат- ся к убегающему на расстояние l>0. В задаче об уклонении мы определили стратегию, гарантирующую уклонение убегающего от преследователя на расстояние большее, чем l>0. Кроме того, показаны новые достаточные условия уклонения.*

Ключевые слова: *дифференциальная игра, l-поимка, убегания, преследова-тель, убегающий, геометрическое ограничение, стратегия, гарантированное время l-поимка.*

Introduction. Differential game problems were first comprehensively looked into by

American mathematician R.Isaacs [1, p. 340] in the 1950th. Subsequently, Pontryagin [2, p. 551] and Krasovsky-Subbotin [3, p. 517] set the fundamental approaches of Theory of Differential Games.

The problem for the case of *l*-approach [5, p. 272] was first proposed and explained by Indian mathematician Ramchundra. Later, Pshenichnyi [6, p. 484-485], Petrosyan [4, p. 31-38], Satimov [7, p. 203], Azamov [8, p. 38-43], Samatov [9, 10] and others studied that problem, and interesting results were obtained by them. In the work of Petrosyan and Dutkevich [4, p. 31-38], the *l*-capture problem was investigated for the players moving at the limited velocities by coordinates on the plane and also, a lifeline game was solved by geometrical method. Later on, by virtue of Chikrii's method of resolving functions, B.T.Samatov [9, p. 907-921; 10, p. 94-107] solved the problem of group pursuit for the case of *l*-capture in a simple motion of the players under integral constraints on controls. In [11, p. 574-579], Khaidarov considered the problem of positional *l*-capture of one evader by a group of pursuers provided that each of the players has a simple movement.

In the paper, we have studied the *l*-capture and evasion problems in a differential game with inertial players whose controls are subject to geometrical constraints. In the *l*-capture problem, an approach strategy is formulated for a pursuer and sufficient conditions of *l*-capture are obtained. In the evasion problem, a specific strategy is suggested for an evader and sufficient conditions of evasion are found. Furthermore, it is shown how the distance between the players changes during the evasion game.

1. Statement of problems. Suppose that in the finite-dementional space $\mathbb{R}^n$, the pursuer chases the evader. Let their motions be based on the differential equations with initial conditions

$$\ddot{x} = u, \quad \dot{x}(0) = x_1, \quad x(0) = x_0, \tag{1}$$

$$\ddot{y} = v, \quad \dot{y}(0) = y_1, \quad y(0) = y_0 \tag{2}$$

correspondingly, where $x, y, u, v \in \mathbb{R}^n$, $n \geq 2$; x_0, y_0 are the initial states of the players for which it is supposed that $|x_0 - y_0| > l$, $l > 0$; x_1, y_1 are their initial velocity vectors accordingly, and assume $x_1 = y_1$; the acceleration vectors u, v function as control parameters of the players, respectively and they depend on the time $t \geq 0$.

The controls u, v are single out as measurable functions $u(\cdot): \mathbb{R}^+ \to \mathbb{R}^n$ and $v(\cdot): \mathbb{R}^+ \to \mathbb{R}^n$, and they are subject to the constraints

$$|u(t)| \leq \alpha \text{ for almost every } t \geq 0, \tag{3}$$

$$|v(t)| \leq \beta \text{ for almost every } t \geq 0, \tag{4}$$

which are usually said the *geometric constraints* (in short, G*-constraints*), where α, β are the given positive parametric numbers which express maximal acceleration values of the players appropriately. The family of all the measurable functions satisfying (3) is denoted by $U_{G,l}$, and the family of all the measurable functions satisfying (4) is denoted by $V_{G,l}$.

Definition 1. A measurable function $u(\cdot) \in U_{G,l}$ $\left(v(\cdot) \in V_{G,l}\right)$ is said an *acceptable control of the pursuer* (*of the evader*).

For some $u(\cdot) \in U_{G,l}$ and $v(\cdot) \in V_{G,l}$, from the equations (1), (2) the triplets $\left(x_0, x_1, u(\cdot)\right)$ and $\left(y_0, y_1, v(\cdot)\right)$ describe the trajectories

$$x(t) = x_0 + x_1 t + \int_0^t (t-s)u(s)ds\,,\ y(t) = y_0 + y_1 t + \int_0^t (t-s)v(s)ds$$

of the players.

The chief target of the pursuer is to converge the evader at the distance $l > 0$ (*l*-capture problem), i.e., to attain the relation

$$|x(\eta) - y(\eta)| \le l \tag{5}$$

at some finite time $\eta > 0$. As for the evader struggles to get out of the occurrence of (5) (evasion problem), i.e., to continue the inequality

$$|x(t) - y(t)| > l \tag{6}$$

for all $t \ge 0$ or if it is impossible, to put off the instant η when the inequality (5) holds.

Obviously, for the pursuer, control functions depending only on the time $t \ge 0$ are not adequate to solve the *l*-capture problem, and suitable types of controls should be strategies.

Let us introduce the denotations: $z(t) = x(t) - y(t),\ z(0) = z_0,\ \dot z(0) = z_1$. Then we have $z_0 = x_0 - y_0,\ z_1 = x_1 - y_1$.

Due to (1), (2), come to the unique Cauchy problem in the form

$$\ddot z = u - v\,,\ \dot z(0) = 0\,,\ z(0) = z_0\,. \tag{7}$$

2. The main results.

Definition 2. For $\alpha \ge \beta$**,** we say the function

$$\mathbf{u}(v, z_0) = v - \lambda(v, z_0)\frac{\alpha z_0 + vl}{\alpha + \lambda(v, z_0)l} \tag{8}$$

the *convergence strategy of the pursuer* or Π_l*-strategy* in the differential game (1)-(4), where $\lambda(v, z_0) = \frac{1}{h^2}\left[\langle v, z_0\rangle + \alpha l + \sqrt{\left(\langle v, z_0\rangle + \alpha l\right)^2 + h^2\left(\alpha^2 - |v|^2\right)}\right],\ h^2 = |z_0|^2 - l^2.$

Note that the function $\lambda(v, z_0)$ is generally called the *resolving function*. Below we will provide some important properties for the strategy (8) and the resolving function $\lambda(v, z_0)$.

Proposition 1. If $\alpha \ge \beta$ holds, then the strategy (8) is defined and continuous for any $v,\ |v| \le \beta$, and the equality $|\mathbf{u}(v, z_0)| = \alpha$ holds during the *l*-capture game.

Proposition 2. If $\alpha \ge \beta$ is valid, then the function $\lambda(v, z_0)$ is defined, non- negative and continuous for any $v,\ |v| \le \beta$, and it is bounded as

$$(\alpha - \beta)/(|z_0| - l) \le \lambda(v, z_0) \le (\alpha + \beta)/(|z_0| - l)\,. \tag{9}$$

Definition 3. In *l*-capture (1)-(4), we say that Π_l-strategy (8) *guarantees to occur l-capture* in some time interval $[0, T_l]$ if, for any $v(\cdot) \in V_{G,l}$:

a) some moment $t_* \in [0, T_l]$, occurs $|z(t_*)| \le l$, is likely to be found;

b) in the time interval $[0,\ t_*]$, the inclusion $\mathbf{u}(v, z_0) \in U_{G,l}$ is satisfied,

where the number T_l is called a *guaranteed time of l-capture*.

Theorem 1. Let $\alpha > \beta$. Then Π_l-strategy (8) guarantees to occur *l*-capture on the interval

$[0,T_l]$ in the l-capture problem (1)-(4), where

$$T_l = \sqrt{2\left(|z_0|-l\right)/\left(\alpha-\beta\right)}.$$

Definition 4. We call the control function

$$v_*(t) = -\beta\frac{z_0}{|z_0|} \tag{10}$$

a *strategy of the evader*.

Definition 5. We say that the strategy $v_*(t)$ is *winning* if, for any control $u(\cdot)\in U_{G,l}$, the solution $z(t)$ of

$$\ddot{z} = u(t)-v_*(t),\ \dot{z}(0)=0,\ z(0)=z_0 \tag{11}$$

fulfills the inequality (6) for all t, $t\geq 0$.

Theorem 2. Let $\alpha\leq\beta$. Then in the evasion game (1)-(4), the strategy $v_*(t)$ is winning in the time interval $[0,+\infty)$, and the distance between the pursuer and the evader varies in terms of the function

$$E(t) = \frac{\beta-\alpha}{2}t^2 + |z_0| - l.$$

REFERENCES

1. Isaacs R. Differential games. John Wiley and Sons, New York. 1965, 340 p.
2. Pontryagin L.S. Selected Works. MAKS Press, Moscow. 2014, 551 p.
3. Krasovsky N.N. Game-Theoretical Control Problems/ Subbotin A.I. Springer, New York. 1988, 517 p.
4. Petrosyan L.A. Games with “a Survival Zone”. Occasion *L*-catch / Dutkevich V.G. Vestnik Leningrad State Univ., 1969, Vol.3, № 13, p. 31-38.
5. Nahin P.J. Chases and escapes: The Mathematics of Pursuit and Evasion. Princeton University Press, Princeton. 2012, p. 272.
6. Pshenichnyi B.N. Simple pursuit by several objects. Cybernetics and Systems Analysis, 1976, Vol. 12, № 5, p. 484-485.
7. Satimov N.Yu. Methods for solving the pursuit problem in the Theory of Differential Games. Izd-vo NUUz, Tashkent. 2003. P. 203
8. Azamov A.A. On the quality problem for simple pursuit games with constraint. Serdica Bulgariacae math., 1986, Vol. 12, № 1, p. 38-43.
9. Samatov B.T. Problems of group pursuit with integral constraints on controls of the players. Cybernetics and Systems Analysis, 2013, Vol. 49, № 6, p. 907-921.
10. Samatov B.T. Differential game with a lifeline for the inertial movements of players / Soyibboyev U.B. Ural Mathematical Journal, Vol. 7, № 2, p. 94-107.
11. Khaidarov B.K. Positional *l*-catch in the game of one evader and several pursuers. Prikl. Matem. Mekh., 1984, Vol. 48, № 4, p. 574-579.