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Initial Value Problem for the Nonlinear Stochastic Impulsive Integro-Differential Equations

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Abstract. *This paper examines the Cauchy problem for a nonlinear system of first-order stochastic integro-differential equations with impulse effects. The model under consideration combines Wiener-type stochastic perturbations, nonlocal integral interactions, and impulse state jumps at predetermined points in time. Such systems arise in the mathematical modeling of control processes, population dynamics, economic systems, technical objects, and other dynamic processes subject to random perturbations and instantaneous state changes. The original Cauchy problem is reduced to an equivalent nonlinear stochastic functional-integral equation. A sequence of stochastic approximations is constructed for the resulting operator, and the necessary a priori estimates are obtained in the space of piecewise continuous random processes. Using the principle of contraction mappings, the existence and uniqueness of a solution to the problem under study are proven. The obtained results establish a constructive framework for finding a solution and expand existing methods for studying nonlinear stochastic impulse systems with integral interactions.*

Key words: *Impulsive stochastic integro-differential equations, initial value condition, expectation, variance, successive approximations, existence and uniqueness of solution.*

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Introduction

Impulsive differential equations have many applications in various fields of science and technology (see, [1]–[7]). Therefore, scientific publications on impulsive differential equations are common [8]–[17].

Modern processes occurring in physics, engineering, biology, economics, ecology, and control theory are often subject to random factors and abrupt short-term changes in state (see, [18]–[30]). Stochastic differential equations with impulse effects are widely used to mathematically describe such phenomena. The stochastic component allows for the influence of external random disturbances to be taken into account, while impulse effects model instantaneous jumps in system parameters that arise due to control actions, emergency events, operating mode switches, or other discrete processes. In recent decades, there has been a steady increase in interest in the study of stochastic impulse systems. Considerable attention has been devoted to the existence, uniqueness, stability, and controllability of solutions to such equations. Systems containing integral terms present particular challenges, as they take into account the accumulated effects of the process's history and lead to the emergence of nonlocal dependencies. Stochastic integro-differential models allow for a more adequate description of real-world processes compared to classical differential equations. However, the simultaneous presence of random disturbances, nonlinear integral operators, and impulse responses significantly complicates the analytical study of the corresponding problems. This paper considers the Cauchy problem for a nonlinear system of first-order stochastic integro-differential equations with impulse responses. The system under study contains a Wiener process, nonlinear integral interactions, and impulse state jumps at fixed instants of time. The main idea of the study is to reduce the original problem to an equivalent stochastic functional-integral equation. A sequence of approximations is constructed for the resulting equation, allowing the application of the principle of contraction mappings in the space of piecewise continuous random processes. This approach provides a constructive proof of the existence and uniqueness of a solution. The scientific novelty of this paper lies in the development of an effective framework for studying nonlinear stochastic impulse integro-differential systems and the derivation of sufficient conditions for the unique solvability of the Cauchy problem.

1 Preliminary materials

The materials in this section are taken from the book [31].

Definition 1.1. A process $w(t)$ is called a Wiener process, if it is defined for $t \geq 0$ and is a homogeneous Gaussian process with independent increments for which $w(0) = 0$, $\mathbf{M}w(t) = 0$, $\mathbf{D}w(t) = t$, where $\mathbf{M}w(t)$ is an expectation of the process $w(t)$, $\mathbf{D}w(t)$, is a variance of $w(t)$.

Therefore, the distribution $w(t+h) - w(t)$ coincides with the distribution $w(h)$ and is a normal distribution with mean zero and variance h:

$$\mathbf{P}\left\{a < w(h) < b\right\} = \frac{1}{\sqrt{2\pi h}} \int\limits_a^b e^{-\dfrac{u^2}{2h}} du.$$

Lemma 1.1. *Let the process $\xi(t)$ be defined and continuous with probability equal to one for $t \geq 0$, $\xi(0) = 0$ and let the $\sigma-$algebras $\mathbb{F}_t$ be defined for all $t \geq 0$, and $\mathbb{F}_{t_1} \subset \mathbb{F}_{t_2}$ for $t_1 < t_2$. If the following conditions are satisfied:*

1). For all $t \geq 0$ the quantity $\xi(t)$ is measurable relative to $\mathbb{F}_t$;

2). $\mathbf{M}\left([\xi(t+h) - \xi(t)] \setminus \mathbb{F}_t\right) = 0$ with probability equal to one for all $t \geq 0$;

3). $\mathbf{M}\left([\xi(t+h) - \xi(t)]^2 \setminus \mathbb{F}_t\right) = h$ with probability equal to one for all $t \geq 0$ and $h > 0$.

Then $\xi(t)$ is a Wiener process.

Consider the stochastic integral $\int\limits_0^T f(t)dw(t)$ over a Wiener process. Let us denote the space $\mathbf{H}_2[0,T]$ of continuous functions $f(t)$, defined for all $t \in [0,T]$ and for each t measurable with respect to $\mathbb{F}_t$, for which the integral $\int\limits_0^T f^2(t)d(t)$ is finite with probability equal to unity. Here the formula holds:

$$\int\limits_0^T f(t)dw(t) = \sum_{k=0}^{m-1} f(t_k)\left[w(t_{k+1}) - w(t_k)\right].$$

Consider the integral $\int\limits_0^T g_n(t)dw(t)$ of a sequence of functions $g_n(t) \in \mathbf{H}_2[0,T]$ that converges to some limit $\int\limits_0^T f(t)dw(t)$. Then the following equalities hold:

$$\mathbf{M}\int\limits_0^T f(t)dw(t) = \lim_{n\to\infty} \mathbf{M}\int\limits_0^T g_n(t)dw(t) = 0,$$

$$\mathbf{M}\left[\int\limits_0^T f(t)dw(t)\right]^2 = \lim_{n\to\infty} \mathbf{M}\left[\int\limits_0^T g_n(t)dw(t)\right]^2 = \lim_{n\to\infty}\left[\int\limits_0^T \mathbf{M}g_n^2(t)dt\right]^2 = \int\limits_0^T \mathbf{M}f^2(t)dt < \infty.$$

Lemma 1.2. *Let $f(t)$ be a step function from the class $\mathbf{H}_2[0,T]$ and $\int\limits_0^T \mathbf{M}f^2(t)dt < \infty$. Then the following estimates are valid:*

$$\mathbf{P}\left\{\sup_{0\le t\le T}\left|\int\limits_0^T f(t)dw(t)\right| > a\right\} \le \frac{1}{a^2}\int\limits_0^T \mathbf{M}f^2(t)dt,$$

$$\mathbf{M}\sup_{0\le t\le T}\left|\int\limits_0^T f(t)dw(t)\right|^2 \le 4\mathbf{M}\left[\int\limits_0^T f(t)dw(t)\right]^2 = 4\int\limits_0^T \mathbf{M}f^2(t)dt.$$

Lemma 1.3 (Ito differential)**.** *Let the process $\xi(t)$ have a stochastic differential*

$$d\xi(t) = a(t)dt + b(t)dw(t).$$

And let the function $f(t,x)$ be continuous and have continuous derivatives $f'_t(t,x)$, $f'_x(t,x)$, $f''_{xx}(t,x)$. Then the process $f(t,\xi(t))$ also has a stochastic differential

$$df(t,\xi(t)) = \left[f'_t(t,\xi(t)) + \frac{1}{2}f''_{\xi\xi}(t,\xi(t))b^2(t) + f'_\xi(t,\xi(t))a(t)\right]dt +$$

$$+ f'_\xi(t,\xi(t))b(t)dw(t).$$

Lemma 1.4. *Let the processes $\alpha(t)$, $\beta(t)$ be measurable bounded functions, and for some $0 < L = \mathrm{const}$ the inequality holds*

$$\alpha(t) \leq \beta(t) + L \int\limits_0^t \alpha(s) ds.$$

Then the following estimate takes place:

$$\alpha(t) \leq \beta(t) + L \int\limits_0^t e^{L(t-s)} \beta(s) ds.$$

2 Problem statement and reduction to a functional integral equation

On the segment $[0,T]$ we consider the following first order stochastic system of nonlinear integro-differential equations

$$dx(t) = f\left(t, x(t), \int\limits_0^T \Theta\left(t, s, x(s)\right) ds\right) dt + g(t, x(t)) dw(t), \ \ t \neq t_i, \ \ i = 1, 2, ..., p \tag{2.1}$$

with initial value condition

$$x(0) = \varphi_0 = \mathrm{const} \tag{2.2}$$

and nonlinear impulsive effect conditions

$$x\left(t_i^+\right) - x\left(t_i^-\right) = I_i\left(x\left(t_i\right)\right), \quad i = 1, 2, ..., p, \tag{2.3}$$

where $0 = t_0 < t_1 < ... < t_p < t_{p+1} = T$, $\Theta(t,s,x) \in \mathbf{H}_2\big([0,T]^2 \times \mathbb{R}^n, \mathbb{R}^n\big)$ is given $n \times n-$dimensional matrix function and $f(t,x,y) \in \mathbf{H}_2([0,T] \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n)$, $g(t,x) \in \mathbf{H}_2\big([0,T] \times \mathbb{R}^n, \mathbb{R}^n\big)$, $\Theta(t,s,x) \in \mathbf{H}_2\big([0,T]^2 \times \mathbb{R}^n, \mathbb{R}^n\big)$, $I_i : \mathbb{R}^n \to \mathbb{R}^n$ are given functions; $x\left(t_i^+\right)$, $x\left(t_i^-\right)$ are right-hand sided and left-hand sided limits of function $x(t)$ at the point $t = t_i$, respectively.

By $\mathbf{H}_2^{PC}\left([0,T], \mathbb{R}^n\right)$ denoted the linear vector space

$$\mathbf{H}_2^{PC}\left([0,T], \mathbb{R}^n\right) = \left\{x : [0,T] \to \mathbb{R}^n; \ x\left(t\right) \in \mathbf{H}_2\left(\left(t_i, t_{i+1}\right], \mathbb{R}^n\right), \ i = 1, ..., p\right\},$$

where $x\left(t_i^+\right)$ and $x\left(t_i^-\right)$ $(i = 0, 1, ..., p)$ exist and bounded; $x\left(t_i^-\right) = x\left(t_i\right)$.

Problem 2.1. To find the function $x(t) \in \mathbf{H}_2^{PC}\left([0,T], \mathbb{R}^n\right)$, which for all $t \in [0,T]$, $t \neq t_i$, $i = 1, 2, ..., p$ satisfies the integro-differential equation (2.1), initial condition (2.2) and for $t = t_i$ $i = 1, 2, ..., p$, $0 < t_1 < t_2 < ... < t_p < T$ satisfies the nonlinear limit conditions (2.3).

Let the function $x(t) \in \mathbf{H}_2^{PC}\left([0,T], \mathbb{R}^n\right)$ is a solution of the initial value problem (2.1)–(2.3). Then by integration of the equation (2.1) on the interval $t \in \left(0, t_{i+1}\right]$, we obtain

$$\int\limits_0^t f(s,x,y) ds + \int\limits_0^t g(s,x) dw(s) = \int\limits_0^t dx(s) ds =$$

$$= \left[x\left(t_1\right) - x\left(0^+\right)\right] + \left[x\left(t_2\right) - x\left(t_1^+\right)\right] + ... + \left[x(t) - x\left(t_i^+\right)\right] =$$
$$= -x(0) - \left[x\left(t_1^+\right) - x\left(t_1\right)\right] - \left[x\left(t_2^+\right) - x\left(t_2\right)\right] - ... - \left[x\left(t_i^+\right) - x\left(t_i\right)\right] + x(t).$$

Taking into account the conditions (2.2) and (2.3), the last equality we rewrite as

$$x(t) = \mathbf{J}(t;x) \equiv \varphi_0 + \int_0^t f\left(t, x(s), \int_0^T \Theta\left(s,\theta,x(\theta)\right) d\theta\right) ds +$$
$$+ \int_0^t g(s,x(s))dw(s) + \sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right). \tag{2.4}$$

It is easy to verify that the equation (2.4) satisfies the problem (2.1)–(2.3).

3 The questions of one value solvability

Theorem 3.1. *Suppose the following conditions are fulfilled:*

1). $N_f = \mathbf{M}\left[f\left(t, x(t), \int\limits_0^T \Theta\left(t,s,x(s)\right) ds\right)\right]^2 < \infty;$

2). $N_g = \mathbf{M}\left[g\left(t,x(t)\right)\right]^2 < \infty;$

3). $m_I = \max\limits_{i\in\{1,2,\dots,p\}} \mathbf{M}\left[I_i\left(x(t)\right)\right]^2 < \infty;$

4). *For all* $t \in [0,T]$, $x, y \in \mathbb{R}^n$ *holds*

$$\left| f\big(t,x_1,y_1\big) - f\big(t,x_2,y_2\big) \right| \le K_1(t)\left(\left| x_1 - x_2 \right| + \left| y_1 - y_2 \right|\right), \quad 0 < K_1(t) \in \mathbf{H}_2[0,T];$$

5). *For all* $t \in [0,T]$, $x \in \mathbb{R}^n$ *holds*

$$\left| g(t,x_1) - g(t,x_2) \right| \le K_2(t)\ \left| x_1 - x_2 \right|, \quad 0 < K_2(t) \in \mathbf{H}_2[0,T];$$

6). *For all* $t, s \in [0,T]^2$, $x \in \mathbb{R}^n$ *holds*

$$\left|\Theta(t,s,x_1) - \Theta(t,s,x_2)\right| \le K_3(t,s)\ \left| x_1 - x_2 \right|, \quad 0 < K_3(t) \in \mathbf{H}_2[0,T]^2;$$

7). *For all* $x \in \mathbb{R}^n$, $i = 0,1,...,p$ *holds*

$$\left| I_i(x_1) - I_i(x_2) \right| \le m_i\ \left| x_1 - x_2 \right|, \quad 0 < m_i = \mathrm{const};$$

8). $x(0)$ *does not depend on* $w(t)$ *and* $\mathbf{M}x^2(0) < \infty$;

9). $\rho = 3\max\limits_{0\le t\le T}\left\{\left| t\int\limits_0^t \left(K_1^2(s) + \int\limits_0^T K_3^2(s,\theta)d\theta\right) ds \right| + \left| \int\limits_0^t K_2^2(s)ds \right| + p\sum_{i=1}^p m_i\right\} < 1.$

Then the initial value problem (2.1)–(2.3) *has a unique solution* $x(t) \in \mathbf{H}_2^{PC}\left([0,T],\mathbb{R}^n\right)$ *with equal to one probability satisfying the conditions:* $x(t)$ *with equal to one probability continuously,* $x(t) = x(0)$ *at* $t = 0$; $\sup\limits_{0\le t\le T} \mathbf{M}x^2(t) < \infty$. *This solution can be found by the following iterative process:*

$$\begin{cases} x_k\left(t\right) = \mathbf{J}(t;x_{k-1}), \quad k = 1,2,3,... \\ x_0\left(t\right) = \varphi_0, \quad t \in (t_i,t_{i+1}), \quad i = 0,1,2,...,p. \end{cases} \tag{3.1}$$

Proof. We consider the following operator

$$\mathbf{J} : \mathbf{H}_2^{PC}\left([0,T];\mathbb{R}^n\right) \to \mathbf{H}_2^{PC}\left([0,T]\times\mathbb{R}^n\right),$$

defined by the right-hand side of equation (2.4). Appling the principle of contracting operators to (2.4), we show that the operator $\mathbf{J}$ has a unique stochastic fixed point. So, we consider the stochastic successive approximations (3.1):

$$x_{k+1}(t) = \varphi_0 + \int\limits_0^t f\left(t, x_k(s), \int\limits_0^T \Theta\left(s,\theta,x_k(\theta)\right)d\theta\right)ds +$$

$$+\int\limits_0^t g(s,x_k(s))dw(s) + \sum_{0<t_i<t} I_i\left(x_k\left(t_i\right)\right), \quad k = 0,1,2,3,...$$

We find the mathematical expectation of successive approximations

$$\mathbf{M}x_{k+1}^2(t) \leq 4\mathbf{M}x_0^2(0) + 4\mathbf{M}\left[\int\limits_0^t f\left(t, x_k(s), \int\limits_0^T \Theta\left(s,\theta,x_k(\theta)\right)d\theta\right)ds\right]^2 +$$

$$+4\mathbf{M}\left[\int\limits_0^t g(s,x_k(s))dw(s)\right]^2 + 4\mathbf{M}\left[\sum_{0<t_i<t} I_i\left(x_k\left(t_i\right)\right)\right]^2 \leq$$

$$\leq 4\mathbf{M}\varphi_0^2 + 4tN_f + 4tN_g + 4pm_I < \infty. \tag{3.2}$$

For the difference between two adjacent successive approximations, we obtain the estimate

$$\mathbf{M}\left[x_{k+1}(t) - x_k(t)\right]^2 = \mathbf{M}\left[\int\limits_0^t \left(f\left(t, x_k(s), \int\limits_0^T \Theta\left(s,\theta,x_k(\theta)\right)d\theta\right) -\right.\right.$$

$$\left.\left. -f\left(t, x_{k-1}(s), \int\limits_0^T \Theta\left(s,\theta,x_{k-1}(\theta)\right)d\theta\right)\right)ds +\right.$$

$$\left. +\int\limits_0^t \left(g(s,x_k(s)) - g(s,x_{k-1}(s))\right)dw(s) + \sum_{0<t_i<t}\left(I_i\left(x_k\left(t_i\right)\right) - I_i\left(x_{k-1}\left(t_i\right)\right)\right)\right]^2 \leq$$

$$\leq 3\mathbf{M}\left[\int\limits_0^t \left(f\left(t, x_k(s), \int\limits_0^T \Theta\left(s,\theta,x_k(\theta)\right)d\theta\right) - f\left(t, x_{k-1}(s), \int\limits_0^T \Theta\left(s,\theta,x_{k-1}(\theta)\right)d\theta\right)\right)ds\right]^2 +$$

$$+3\mathbf{M}\left[\int\limits_0^t \left(g(s,x_k(s)) - g(s,x_{k-1}(s))\right)dw(s)\right]^2 +$$

$$+3\mathbf{M}\left[\sum_{i=1}^p \left(I_i\left(x_k\left(t_i\right)\right) - I_i\left(x_{k-1}\left(t_i\right)\right)\right)\right]^2 \leq$$

$$\leq 3\mathbf{M}t\int\limits_0^t\left[f\left(t,x_k(s),\int_0^T\Theta\left(s,\theta,x_k(\theta)\right)\,d\theta\right)-f\left(t,x_{k-1}(s),\int\limits_0^T\Theta\left(s,\theta,x_{k-1}(\theta)\right)d\theta\right)\right]^2 ds+$$

$$+3\mathbf{M}\int\limits_0^t\left[g(s,x_k(s))-g(s,x_{k-1}(s))\right]^2 ds+$$

$$+3p\mathbf{M}\sum_{i=1}^p\left[I_i\left(x_k\left(t_i\right)\right)-I_i\left(x_{k-1}\left(t_i\right)\right)\right]^2\leq$$

$$\leq 3\max_{0\leq t\leq T}\left|t\int\limits_0^t\left(K_1^2(s)+\int\limits_0^T K_3^2(s,\theta)d\theta\right)ds\right|\mathbf{M}\left[x_k(s)-x_{k-1}(s)\right]^2+$$

$$+3\max_{0\leq t\leq T}\left|\int\limits_0^t K_2^2(s)ds\right|\mathbf{M}\left[x_k(s)-x_{k-1}(s)\right]^2+$$

$$+3p\sum_{i=1}^p m_i\mathbf{M}\left[x_k(s)-x_{k-1}(s)\right]^2\leq\rho\cdot\mathbf{M}\left[x_k(s)-x_{k-1}(s)\right]^2, \tag{3.3}$$

where

$$\rho=3\max_{0\leq t\leq T}\left\{\left|t\int\limits_0^t\left(K_1^2(s)+\int\limits_0^T K_3^2(s,\theta)d\theta\right)ds\right|+\left|\int\limits_0^t K_2^2(s)ds\right|+p\sum_{i=1}^p m_i\right\}.$$

According to the last condition of the theorem $\rho<1$. Therefore, from the estimate (3.3) we have

$$\mathbf{M}\left[x_{k+1}(t)-x_k(t)\right]^2<\mathbf{M}\left[x_k(t)-x_{k-1}(t)\right]^2. \tag{3.4}$$

It follows from (3.4) that the stochastic operator J on the right-hand side of the equation (2.4) is contracting. According to fixed point principle, taking into account estimates (3.2)–(3.4), we conclude that the operator $\mathbf{J}$ has a unique stochastic fixed point. Consequently, the initial value problem (2.1)–(2.3) has a stochastic unique solution $x(t)\in\mathbf{H}_2^{PC}\left([0,T],\mathbb{R}^n\right).$ □

Remark 3.1. If in conditions (2.3) $I_i\left(x\left(t_i\right)\right)=0,\ i=1,2,...,p$, then the existence of a solution to the initial problem is reduced to estimating the iteration by the factorial law! And the proof of the uniqueness of the solution is reduced to using Lemma 1.4. Let $x(t)$ and $y(t)$ be two solutions to equation (2.1), then for their difference we have the estimate

$$\mathbf{P}\left\{\sup_{0\leq t\leq T}|\,x(t)-y(t)\,|=0\right\}=1.$$

Conclusion

This paper examines the Cauchy problem for a nonlinear system of first-order stochastic integro-differential equations with impulse effects. The model under consideration combines three important mechanisms: stochastic perturbations, the integral memory of the system, and instantaneous impulse state changes. It is shown that the original problem is equivalent to a certain nonlinear stochastic functional-integral equation. An iterative process of successive approximations is constructed for the resulting operator, and the necessary estimates are obtained to establish its contraction properties. Based on the Banach fixed-point principle, the existence and uniqueness of a solution to the Cauchy problem in the space of piecewise continuous random processes are proven. The proposed approach is constructive and allows not only to justify the solvability of the problem but also to construct a sequence of approximations converging to the desired solution. The obtained results expand the theory of stochastic impulse integro-differential equations and can serve as a basis for further research into stability problems, optimal control, stochastic systems with delay, fractional stochastic models, and multidimensional impulse processes. Developments in these areas are of significant interest from both theoretical and applied perspectives.

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